Math 151, Spring 2011
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Quiz # 2

Name: ________________________________

1. Consider the following distributions (conditional and marginal, respectively):

$$\begin{aligned} f_{Y|X}(y|x) &= \lambda e^{-\lambda(y-x)} && y \geq x \\ f_X(x) &= \lambda e^{-\lambda x} && x \geq 0 \end{aligned}$$

Are X & Y independent? Justify your answer, and show your work.

We know that regardless of independence, the joint probability is the product of the conditional and the marginal:

$$\begin{aligned} f(x,y) &= f(y|x) f_X(x) \\ &= \lambda^2 e^{-\lambda y} \qquad y \geq x \end{aligned}$$

Using the joint distribution of X and Y, we can find the marginal distribution of Y:

$$\begin{aligned} f_Y(y) &= \int_0^y f(x,y)dx \\ &= \int_0^y \lambda^2 e^{-\lambda y} dx \\ &= \lambda^2 e^{-\lambda y} y \\ &\neq f(y|x) \end{aligned}$$

So X and Y are not independent.

2. Consider the following joint density (pdf) of X and Y:

$$f_{XY}(x,y) = \frac{12}{7}(x^2 + xy) \qquad 0 \leq x \leq 1, \quad 0 \leq y \leq 1$$

Find the cumulative distribution function (cdf) of Y: $F_Y(y)$

$$\begin{aligned} f(x,y) &= \frac{12}{7}(x^2 + xy) \qquad 0 \leq x \leq 1, \quad 0 \leq y \leq 1 \\ f_Y(y) &= \int_0^1 \frac{12}{7}(x^2 + xy)dx \\ &= \frac{12}{7}\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 y\right|_0^1 \\ &= \frac{12}{7}\left(\frac{1}{3} + \frac{1}{2}y\right) \\ F_y(y) &= \frac{12}{7}\int_0^y \left(\frac{1}{3} + \frac{1}{2}t\right)dt \\ &= \frac{12}{7}\left(\frac{1}{3}t + \frac{1}{2}\frac{1}{2}t^2\right|_0^y \\ &= \frac{12}{7}\left(\frac{1}{3}y + \frac{1}{4}y^2\right) \\ &= \frac{1}{7}(4y + 3y^2) \qquad 0 \leq y \leq 1 \end{aligned}$$