Math 151, Spring 2011
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Quiz # 3

Name: _____

1. Consider a random circle determined by a radius random variable (R).

$$f_R(r) = \lambda e^{-\lambda r} \qquad r > 0$$

Find the pdf (density) for the random variable which is the area of the circle. (Recall that area is π times radius squared.)

Let $A = \pi R^2$.

$$\begin{aligned} F_A(a) &= P(A \leq a) = P(\pi R^2 \leq a) \\ &= P(R^2 \leq a/\pi) \\ &= P(R \leq \sqrt{a/\pi}) \quad \text{note: } R > 0 \\ &= F_R(\sqrt{a/\pi}) \end{aligned}$$

$$\begin{aligned} f_A(a) &= \frac{\partial F_R(\sqrt{a/\pi})}{\partial a} \\ &= f_R(\sqrt{a/\pi}) \frac{1}{2\sqrt{\pi}} a^{-1/2} \\ &= \frac{\lambda}{2\sqrt{\pi}} a^{-1/2} e^{-\lambda\sqrt{\frac{a}{\pi}}} \quad a > 0 \end{aligned}$$

2. You have two instruments with which to measure the height of a tower. The true height is 100 meters, measurements with the first instrument vary with mean 100 meters and standard deviation 1.2 meters. Measurements with the second instrument vary with mean 100 meters and standard deviation 0.85 meters. You make one measurement with each instrument. Your results are X_1 for the first and X_2 for the second, and they are independent.

   (a) To combine the two measurements, you first average them, $Y_1 = \frac{X_1+X_2}{2}$. What are the mean and standard deviation of Y_1? Show your work.

$$\begin{aligned} E[Y_1] &= E[0.5(X_1 + X_2)] \\ &= 0.5[E(X_1) + E(X_2)] = 100 \end{aligned}$$

$$\begin{aligned} Var[Y_1] &= Var[0.5(X_1 + X_2)] \\ &= 0.25[Var(X_1) + Var(X_2)] \\ &= 0.25[1.2^2 + 0.85^2] \\ &= 0.541 \\ SD[Y_1] &= \sqrt{0.541} = 0.735 \end{aligned}$$

   (b) To combine the two measurements, you might take a weighted average of them, $Y_2 = \frac{X_1+2*X_2}{3}$. What are the mean and standard deviation of Y_2? Show your work.

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\begin{aligned}
E[Y_2] &= E[(X_1 + 2 * X_2)/3] \\
&= (1/3)[E(X_1) + 2 * E(X_2)] = 100 \\
\\
Var[Y_1] &= Var[(X_1 + 2 * X_2)/3] \\
&= (1/9)[Var(X_1) + 4 * Var(X_2)] \\
&= (1/9)[1.2^2 + 4 * 0.85^2] \\
&= 0.481 \\
SD[Y_1] &= \sqrt{0.481} = 0.694
\end{aligned}
$$

(c) Which value should you report, Y_1 or Y_2? Explain.

Y_2: It would be better to take a weighted average (weighting X_2 more) because X_2 is less variable that X_1 (making Y_2 less variable than Y_1). On average, we'll get the right expected value with either measure, but our results will be less variable with Y_2.