Math 158, Spring 2013
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Multiple Regression III – R code
Multicollinearity

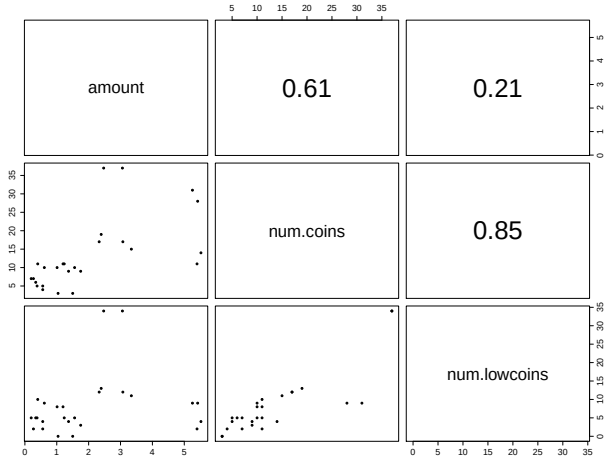
Consider the multiple regression model:

$$\begin{aligned} E[Y] &= \beta_0 + \beta_1 X_1 + \beta_2 X_2 \\ Y &= \text{amount of money in pocket} \\ X_1 &= \#\text{ of coins in pocket} \\ X_2 &= \#\text{ of pennies, nickels, dimes in pocket} \end{aligned}$$

Using a completely non-random sample, I got the following data:

```
> amount <- c(1.37, 1.01, 1.5, 0.56, 0.61, 3.06, 5.42, 1.75, 5.4, 0.56,
              0.34, 2.33, 3.34)
> num.coins <- c(9,10,3,5,10,37,28,9,11,4,6,17,15)
> num.lowcoins <- c(4,8,0,4,9,34,9,3,2,2,5,12,11)

> summary( lm(amount ~ num.coins) )
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.73222    0.66848   1.095   0.2968
num.coins    0.10812    0.04237   2.552   0.0269 *

> summary( lm(amount ~ num.lowcoins) )
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.73038    0.67992   2.545   0.0272 *
num.lowcoins  0.04617    0.05909   0.781   0.4512

> coin.lm <- lm(amount ~ num.coins + num.lowcoins)
> summary(coin.lm)
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.30724    0.46569   0.660 0.524321
num.coins     0.29648    0.05778   5.132 0.000443 ***
num.lowcoins -0.24629    0.06561  -3.754 0.003762 **

Residual standard error: 0.9781 on 10 degrees of freedom
Multiple R-squared: 0.7392,     Adjusted R-squared: 0.6871
F-statistic: 14.17 on 2 and 10 DF,  p-value: 0.001206
```

```
> anova(coin.lm)
Analysis of Variance Table

Response: amount
             Df  Sum Sq Mean Sq F value   Pr(>F)
num.coins     1 13.6423 13.6423  14.260 0.003624 **
num.lowcoins  1 13.4794 13.4794  14.089 0.003762 **
Residuals    10  9.5670  0.9567
```



Effects of Multicollinearity

In reality, there is always some degree of correlation between the explanatory variables (pg 283). for regression models, it is important to understand the entire context of the model, particularly for correlated variables.

1. Regardless of the degree of multicollinearity, our ability to obtain a good fit and make predictions (mean or individual) is not inhibited.

2. If the variables are highly correlated, many different linear combinations of them will produce equally good fits. That is, different samples from the same population may produce wildly different estimated coefficients. For this reason, the variability associated with the coefficients can be quite high. Additionally, the explanatory variables can be statistically not significant even though a definite relationship exists between the response and the set of predictors.

3. We can no longer interpret the coefficient to mean "the change in response when this variable increases by one unit and the others are held constant" because it may be impossible to hold the other variables constant. The regression coefficients do not reflect any inherent effect of the particular predictor variable on the response but rather a marginal or partial effect given whatever other correlated predictor variables are included in the model.