Let μ_{ij} be the **population mean response** for a given treatment in a two-factor study where i refers to the level of factor $A (i = 1, \ldots, a)$ and j refers to the level of factor $B (j = 1, \ldots, b)$. Note: we assume equal sample sizes ($n = n_T/(a \cdot b)$) in each group.

The **row averages** are the A factor level means, the **column averages** are the B factor level means. In general:

$$\mu_{.j} = \frac{\sum_{i=1}^{a} \mu_{ij}}{a} \qquad \mu_{i.} = \frac{\sum_{j=1}^{b} \mu_{ij}}{b}$$

The **overall mean** is denoted by $\mu_{..}$ and is defined as follows:

$$\mu_{..} = \frac{\sum_i \sum_j \mu_{ij}}{ab} = \frac{\sum_i \mu_{i.}}{a} = \frac{\sum_j \mu_{.j}}{b}$$

The **main effects** are defined as the difference between each factor level mean and the overall mean. (E.g., how different is the average effect of level i of A from the overall average?)

$$\alpha_i = \mu_{i.} - \mu_{..} \qquad \beta_j = \mu_{.j} - \mu_{..}$$

Note: in order to be able to estimate the parameters, we require: $\sum_i \alpha_i = 0$ and $\sum_j \beta_j = 0$.

For a model with **Additive Factor Effects** we have:

$$\mu_{ij} = \mu_{..} + \alpha_i + \beta_j \quad (= \mu_{i.} + \mu_{.j} - \mu_{..})$$

For a model with **Interactive Effects** we have:

$$\mu_{ij} = \mu_{..} + \alpha_i + \beta_j + \alpha\beta_{ij} \text{ (where } \sum_i \alpha\beta_{ij} = \sum_j \alpha\beta_{ij} = 0)$$

Source	SS	df	MS	F
Between	$n \sum_i \sum_j (\overline{Y}_{ij.} - \overline{Y}_{...})^2$	ab - 1	MSTR	MSTR/MSE
A	$nb \sum_i (\overline{Y}_{i..} - \overline{Y}_{...})^2$	a-1	MSA	MSA/MSE
B	$na \sum_j (\overline{Y}_{.j.} - \overline{Y}_{...})^2$	b-1	MSB	MSB/MSE
AB	$n \sum_i \sum_j (\overline{Y}_{ij.} - \overline{Y}_{i..} - \overline{Y}_{.j.} + \overline{Y}_{...})^2$	(a-1)(b-1)	MSAB	MSAB/MSE
Error	$\sum_i \sum_j \sum_k (Y_{ijk} - \overline{Y}_{ij.})^2$	$n_T - ab = ab(n-1)$	MSE	
Total	$\sum_i \sum_j \sum_k (Y_{ijk} - \overline{Y}_{...})^2$	$n_T - 1$		

Expected Mean Squares

$$\begin{aligned}
E(MSA) &= \sigma^2 + \frac{nb \sum_i \alpha_i^2}{(a-1)} = \sigma^2 + \frac{nb \sum_i (\mu_{i.} - \mu_{..})^2}{(a-1)} \\
E(MSB) &= \sigma^2 + \frac{na \sum_j \beta_j^2}{(b-1)} = \sigma^2 + \frac{na \sum_j (\mu_{.j} - \mu_{..})^2}{(b-1)} \\
E(MSAB) &= \sigma^2 + \frac{n \sum_i \sum_j (\alpha\beta_{ij})^2}{(a-1)(b-1)} = \sigma^2 + \frac{n \sum_i \sum_j (\mu_{ij} - \mu_{i.} - \mu_{.j} + \mu_{..})^2}{(a-1)(b-1)} \\
E(MSE) &= \sigma^2
\end{aligned}$$