Math 158 – Spring 2009
Jo Hardin
warm-up # 11

Name: _____

In a two-factor study, the treatment means, μ_{ij}, are as follows:

	Factor B		
Factor A	B_1	B_2	B_3
A_1	34	23	36
A_2	40	29	42

1. Obtain the main effects of factor A (α_i). What do these values represent?
2. Does the fact that $\mu_{12} - \mu_{11} = -11$ while $\mu_{13} - \mu_{12} = 13$ establish that factors A and B interact? If yes, explain. If no, explain whether it is possible to establish that the factors A and B interact.

Solution:

1.
$$\begin{aligned}
\mu_{..} &= \frac{\sum_i \sum_j \mu_{ij}}{ab} \\
&= \frac{34+23+36+40+29+42}{6} = 34 \\
\mu_{1.} &= \frac{\sum_j \mu_{1j}}{b} \\
&= \frac{34+23+36}{3} = 31 \\
\mu_{2.} &= \frac{\sum_j \mu_{2j}}{b} \\
&= \frac{40+29+42}{3} = 37 \\
\alpha_1 &= \mu_{1.} - \mu_{..} = 31 - 34 = -3 \\
\alpha_2 &= \mu_{2.} - \mu_{..} = 37 - 34 = 3
\end{aligned}$$

α_1 represents the difference between the true mean for group 1 and the overall population mean. α_2 represents the difference between the true mean for group 2 and the overall population mean.

2. $\alpha\beta_{ij} = \mu_{ij} - \mu_{..} - \alpha_i - \beta_j$. In order to establish that factors A and B interact, we have to show that any of the $\alpha\beta_{ij} \neq 0$.

   Note that:

$$\begin{aligned}
\mu_{12} - \mu_{11} &= \beta_2 - \beta_1 + \alpha\beta_{12} - \alpha\beta_{11} \\
\mu_{13} - \mu_{12} &= \beta_3 - \beta_2 + \alpha\beta_{13} - \alpha\beta_{12}
\end{aligned}$$

   So, no, the above information does not establish that A and B interact (we need more information about the main effects due to B). In order to establish that A and B interact, we need to show that $\alpha\beta_{ij} \neq 0$.

$$\begin{aligned}
\mu_{.1} &= \frac{\sum_i \mu_{i1}}{a} \\
&= \frac{34+40}{2} = 37 \\
\beta_1 &= \mu_{.j} - \mu_{..} = 34 - 34 = 3 \\
\alpha\beta_{11} &= \mu_{11} - \mu_{..} - \alpha_1 - \beta_1 \\
&= 34 - 34 - (-3) - 3 = 0
\end{aligned}$$

   It turns out that $\alpha\beta_{ij} = 0 \ \ \forall \ \ i, j$, so the factors A and B do not interact.