Math 158 – Spring 2009
Jo Hardin
warm-up # 1

Name: _____

Given the model with an interaction term:

$$\begin{aligned} E[Y] &= \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 \cdot X_2 \\ Y &= \text{baby's weight in ounces} \\ X_1 &= \text{lbs mother gained} \\ X_2 &= \text{smoking status} \end{aligned}$$

1. Describe what σ measures in the above model.
2. How is SSE calculated for these data?
3. How is MSE related to σ^2?

Solution:

1. σ (or σ^2) represents the variability of the observed values around the mean value from which the response was drawn. That is, considering the linear hyperplane of explanatory variables (in this case, 2 different simple linear regression lines), the observed response values will vary according to σ around the hyperplane at the given explanatory values. (Note that the above description is identical to saying that σ^2 represents the variance of the model error terms.)
2. $$\begin{aligned} \text{SSE} &= \sum_i (Y_i - \hat{Y}_i)^2 \\ \text{MSE} &= \frac{SSE}{n-p} = \frac{\sum_i (Y_i - \hat{Y}_i)^2}{n-4} \end{aligned}$$
3. MSE is the estimated value of variance of the model error terms. That is, MSE is the statistic that estimates the parameter, σ^2.