Math 159 – Spring 2010
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warm-up # 1

Name: ________________________________

In class I discussed the equivalency of the exact binomial hypothesis test and confidence interval given below:

HT

$$H_0 : p = p^*$$
$$H_1 : p \neq p^*$$

CI $[p_1^*, p_2^*]$ such that

$$P(Y \geq y | p = p_1^*) = \frac{\alpha}{2} = \sum_{i=y}^{n} (p_1^*)^i (1 - p_1^*)^{n-i}$$
$$P(Y \leq y | p = p_2^*) = \frac{\alpha}{2} = \sum_{i=0}^{y} (p_2^*)^i (1 - p_2^*)^{n-i}$$

1. What does it mean for those items to be equivalent? (This part is intuition.)
2. How do we know that the two items are equivalent? (This part is mathematical.)
3. What can you say about the error rate(s) (α)?

Solution:

Let's say that $p^* \notin [p_1^*, p_2^*]$. Without loss of generality, $p^* > p_2^*$. Because the null value (p^*) is bigger than the observed value ($\hat{p}$, inside the interval), we know that our data or more extreme is represented by $Y \leq y$. That means

$$P(Y \leq y | p^*) < P(Y \leq y | p_2^*) = \frac{\alpha}{2}$$
$$\text{p-value} = 2 \cdot P(Y \leq y | p^*) < \alpha$$

The logic works backwards as well. That is, if p-value $< \alpha$, we will reject the null hypothesis. (Similarly, if p^* is in the CI, we don't reject H_0.)

Note that we reject a true null hypothesis $\alpha 100\%$ of the time. That means that we'll create a CI that doesn't contain the true value exactly $\alpha 100\%$ of the time.