Math 159 – Spring 2010
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warm-up # 2

Name: ________________________________

Consider the following CI for the 0.75 quantile: $[X^{(3)}, X^{(n-1)}]$. Assume X is continuous. What is the probability of selecting a simple random sample that will create a confidence interval which contains the true 0.75 quantile ($x_{0.75}$)?

Solution:

$$\begin{aligned}
P(x_{0.75} \notin [X^{(3)}, X^{(n-1)}]) &= P(x_{0.75} < X^{(3)} \text{ or } x_{0.75} > X^{(n-1)}) \quad \text{disjoint, so add probabilities} \\
P(x_{0.75} < X^{(3)}) &= (0.25)^n + n(0.25)^{n-1}(0.75) + \frac{n(n-1)}{2}(0.25)^{n-2}(0.75)^2 \\
P(x_{0.75} > X^{(n-1)}) &= (0.75)^n + n(0.75)^{n-1}(0.25) \\
P(x_{0.75} \notin [X^{(3)}, X^{(n-1)}]) &= (0.25)^n + n(0.25)^{n-1}(0.75) + \frac{n(n-1)}{2}(0.25)^{n-2}(0.75)^2 + \\
&\quad (0.75)^n + n(0.75)^{n-1}(0.25) \\
&= \sum_{i=0}^{3-1} \binom{n}{i} (0.75)^i (0.25)^{n-i} + \sum_{i=n-1}^{n} \binom{n}{i} (0.75)^i (0.25)^{n-i} \\
P(x_{0.75} \in [X^{(3)}, X^{(n-1)}]) &= 1 - \sum_{i=0}^{3-1} \binom{n}{i} (0.75)^i (0.25)^{n-i} - \sum_{i=n-1}^{n} \binom{n}{i} (0.75)^i (0.25)^{n-i}
\end{aligned}$$