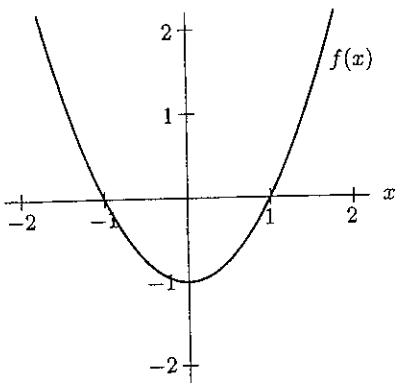
For Problems 1–10, suppose $P_2(x) = a + b(x-1) + c\dfrac{(x-1)^2}{2}$ is a Taylor polynomial of degree two about $x = 1$ for some function f. Give the signs of a, b, and c, if the graph of f is as shown. (Note: 0 is also a possible answer for a and b, but not for c.)

1.



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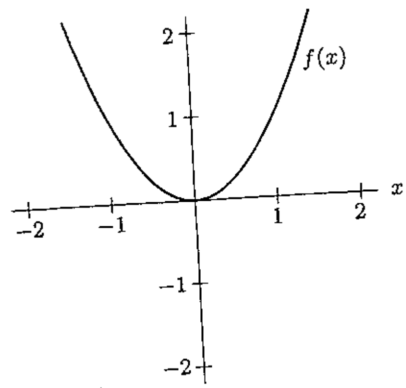

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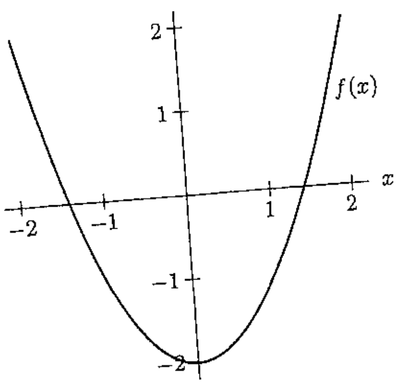

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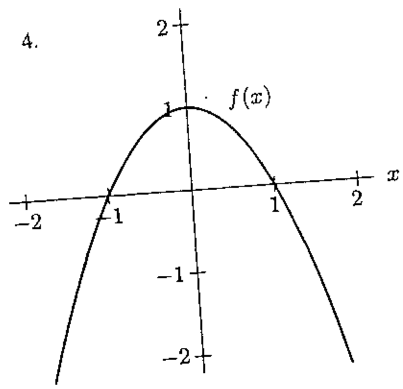

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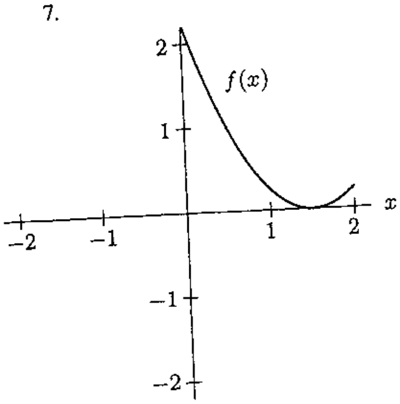

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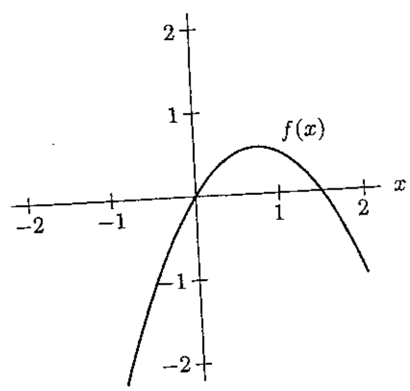


13. The graph in Figu… .4 is generated by what combination of the graphs of the three functions f_1, f_2, and f_3?

(a) $f_1 + f_2 + f_3$ (b) $f_1 - f_2 + f_3$ (c) $-f_1 + f_2 + f_3$
(d) $-f_1 + 0(f_2) - f_3$ (e) $f_1 + 0(f_2) + f_3$ (f) $f_1 + 0(f_2) - f_3$

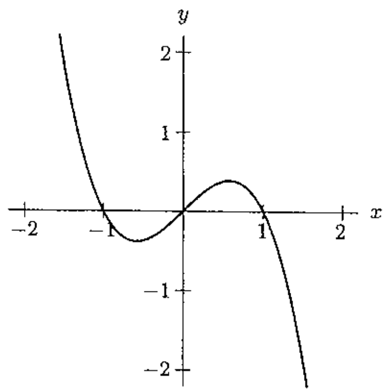
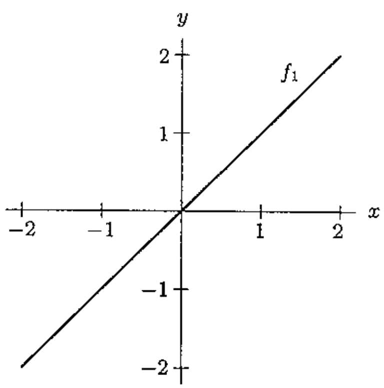
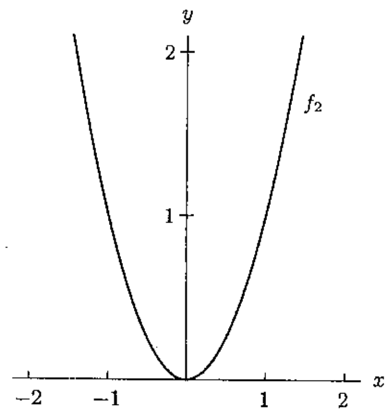
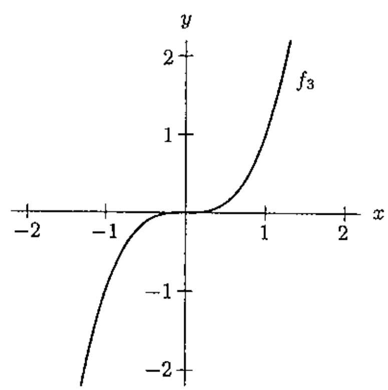


Figure 10.4







14. The graph in Figure 10.5 is generated by what combination of the graphs of the three functions f_1, f_2, and f_3?

(a) $f_1 + f_2 + f_3$ (b) $0(f_1) - f_2 + f_3$ (c) $-f_1 + 0(f_2) + f_3$
(d) $-f_1 + f_2 - f_3$ (e) $f_1 + 0(f_2) - f_3$ (f) $0(f_1) + f_2 + f_3$

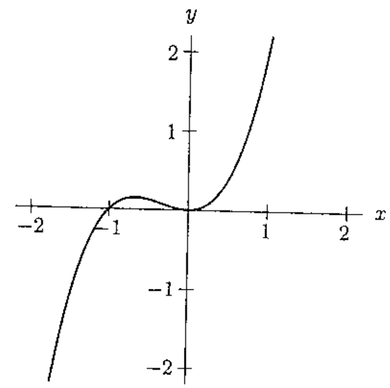
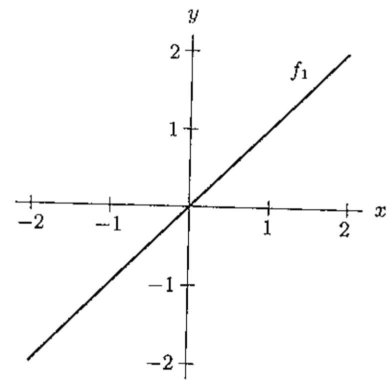
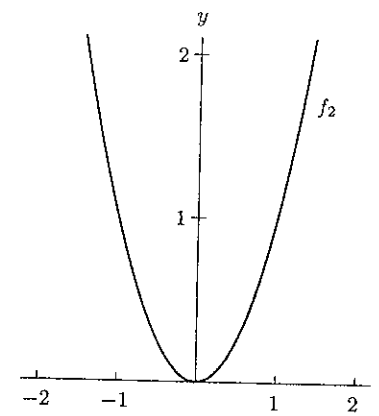
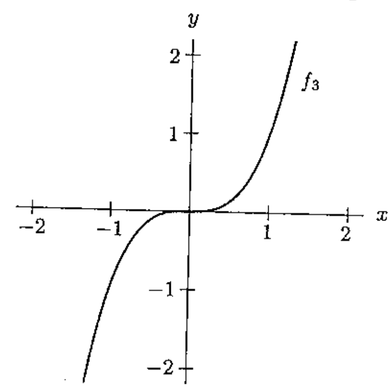


Figure 10.5







11. Which of (a)–(d) is the graph of the addition of the two functions in Figures 10.1 and 10.2?

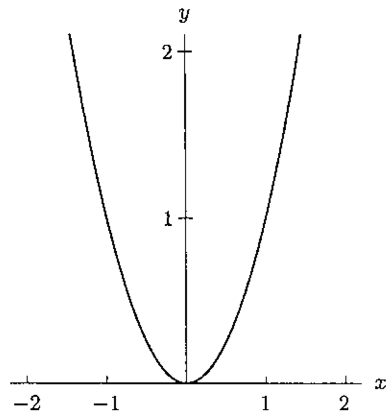


Figure 10.1

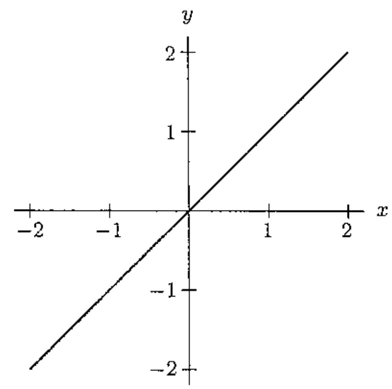
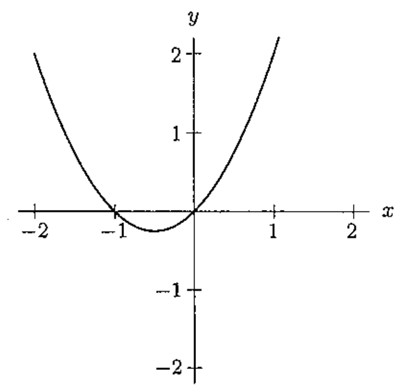
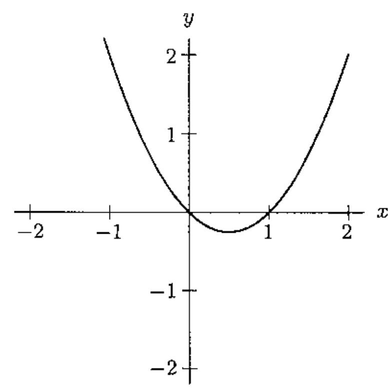


Figure 10.2

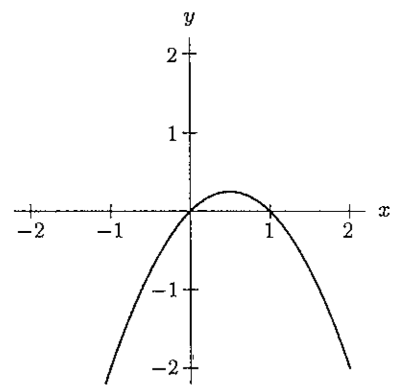
(a)



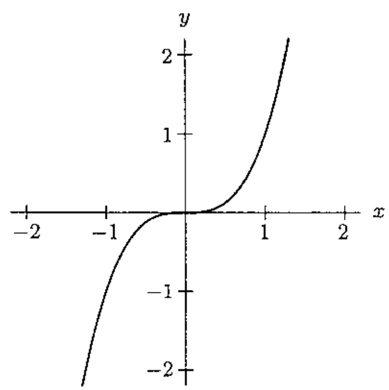
(b)



(c)



(d)



12. The g in Figure 10.3 is generated by what combination of the graphs of the three functions f_1, d f_3?

(a) $f_1 + 0(f_2) + f_3$ (b) $f_1 - f_2 + f_3$ (c) $-f_1 + 0(f_2) + f_3$

(d) $-f_1 + 0(f_2) - f_3$ (e) $f_1 + 0(f_2) - f_3$ (f) $-f_1 + f_2 + f_3$

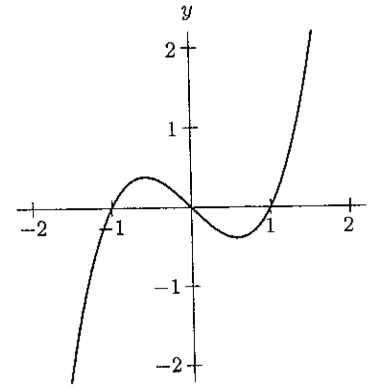
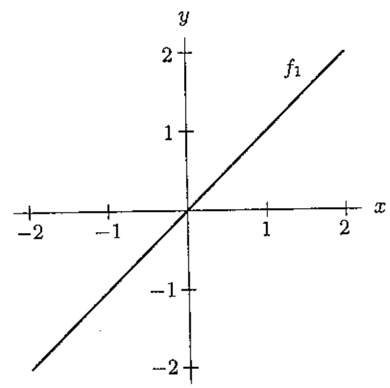
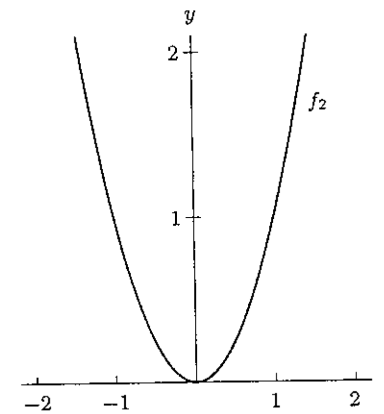
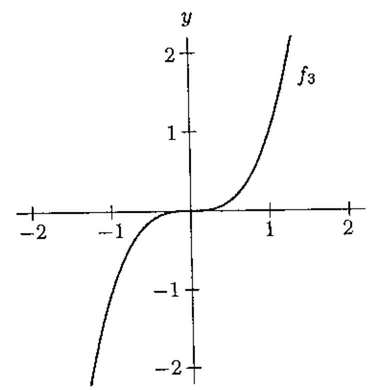


Figure 10.3







1. ANSWER:
0, +, +. The graph contains the point $(1, 0)$ and is increasing and concave up there.
COMMENT:
Follow-up Question. What would be the signs of a, b, and c if the Taylor series was about -1 instead of 1?
Answer. 0, −, +, since the graph contains the point $(-1, 0)$ and is decreasing and concave up when $x = -1$.

2. ANSWER:
+, +, +. At $x = 1$ the function is positive, increasing, and concave up.
COMMENT:
Follow-up Question. What would be the signs of a, b, and c if the Taylor series was about -1 instead of 1?
Answer. +, −, +. At $x = -1$ the function is positive, decreasing, and concave up.

3. ANSWER:
−, +, +. At $x = 1$ the function is negative, increasing, and concave up.
COMMENT:
Follow-up Question. What would be the signs of a, b, and c if the Taylor series was about -1 instead of 1?
Answer. −, −, +. At $x = -1$ the function is negative, decreasing, and concave up.

4. ANSWER:
0, −, −. The graph contains the point $(1, 0)$ and is decreasing and concave down there.
COMMENT:
Follow-up Question. What would be the signs of a, b, and c if the Taylor series was about -1 instead of 1?
Answer. 0, +, −, since the graph contains the point $(-1, 0)$ and is increasing and concave down there.

7. ANSWER:
+, −, +. At $x = 1$ the function is positive, decreasing, and concave up.
COMMENT:
Follow-up Question. What would be the signs of a, b, and c if the Taylor series was about 1.5 instead of 1?
Answer. 0, 0, +. The point $(1.5, 0)$ is on the graph. Also, when $x = 1.5$, the function $f(x)$ has a horizontal tangent and is concave up.

8. ANSWER:
+, −, −. At $x = 1$ the function is positive, decreasing, and concave down.
COMMENT:
Follow-up Question. What would be the signs of a, b, and c if the Taylor series was about 0 instead of 1?
Answer. 0, +, −. The point $(0, 0)$ is on the graph. Also, when $x = 0$, the function $f(x)$ is increasing and concave down.

11. ANSWER:
(a). Both of the original functions are increasing for $x > 0$, so their sum must be also. This eliminates (b) and (c). At $x = -1$, one function has a value 1, while the other has the value -1, so their sum is 0.
COMMENT:
You could also check what happens to the addition of function values for several values of x.
Follow-up Question. What if you subtract the two functions? (You could ask them to do it both ways. They should realize that if both answers are graphed on the same set of axes, then one is a reflection across the x-axis of the other one.)
Answer. Answer (b) is the graph of the difference of the function in Figure 10.1 from the function in Figures 10.2. Answer (c) is the other difference between the two functions.

12. ANSWER:
(c). The graph describes an odd function. This eliminates (b) and (f). The point (1,0) is on the graph which eliminates (a) and (d). The functions is positive for $x > 1$ which eliminates (e).
COMMENT:
It may be worth asking which function, f_1, f_2, or f_3, is not like the graph in Figure 10.3 when the question is first presented to the class. This might encourage your students to think about an odd/even function argument.

13. ANSWER:
(f). The point (1,0) is on the graph which eliminates (a), (b), (c), (d), and (e).
COMMENT:
While the answer gives a quick solution, you may want to spend more time with geometric reasoning. For example, the fact that the graph in Figure 10.4 is an odd function eliminates (a), (b), and (c).

14. ANSWER:
(f). The function is not odd which eliminates (c) and (e). The point $(-1, 0)$ is on the graph which eliminates (a), (b), and (d).
COMMENT:
Notice that $f_3 - f_1$ has a curve that looks somewhat like Figure 10.5 when $x < 0$. However, they are quite different for $x > 0$.