Math 31, Spring 2011
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WU # 4

Name: ______________________________

Suppose you have a variable that has a normal distribution with mean μ and standard deviation σ. You are interested in finding $P(a \leq X \leq b)$.

Using the substitution $z = (x-\mu)/\sigma$, show that the probability of interest is exactly the same as the probability $P((a-\mu)/\sigma \leq Z \leq (b-\mu)/\sigma)$ where Z has a standard normal distribution.

Recall:

$$\begin{aligned} p(x) &= \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \\ p(z) &= \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \end{aligned}$$

Solution:

$$\begin{aligned} P(a \leq X \leq b) &= \int_a^b p(x)dx \\ &= \int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} dx \end{aligned}$$

letting $z = (x-\mu)/\sigma$, $dz = dx/\sigma$.

$$\begin{aligned} P(a \leq X \leq b) &= \int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} dx \\ &= \int_{x=a}^{x=b} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \\ &= \int_{z=(a-\mu)/\sigma}^{z=(b-\mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \\ &= P((a-\mu)/\sigma \leq Z \leq (b-\mu)/\sigma) \end{aligned}$$