Score: ⎯⎯⎯⎯⎯⎯   Name: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Project 1 - Art Gallery Theorem

Math 1030Q – Fall 2014
Professor Hohn

Show all of your work! Write neatly. No credit will be given to unsupported answers. Projects are due *at the beginning of class.* Any project not collected by the instructor at the beginning of class is considered late (and will receive 0 points on the project). No late projects will be accepted!

Part 1: How many guardsmen

1. Draw an art gallery with 13 vertices that needs at least three guards. Prove that only three guards are needed via triangulation of the gallery and correct colorings.

2. Draw an art gallery (other than the 3 crown) with 12 vertices that needs at least 3 guards. Prove that only three guards are needed via triangulation of the gallery and correct colorings.

3. What is the minimal number of guardsmen needed to guard the following gallery:
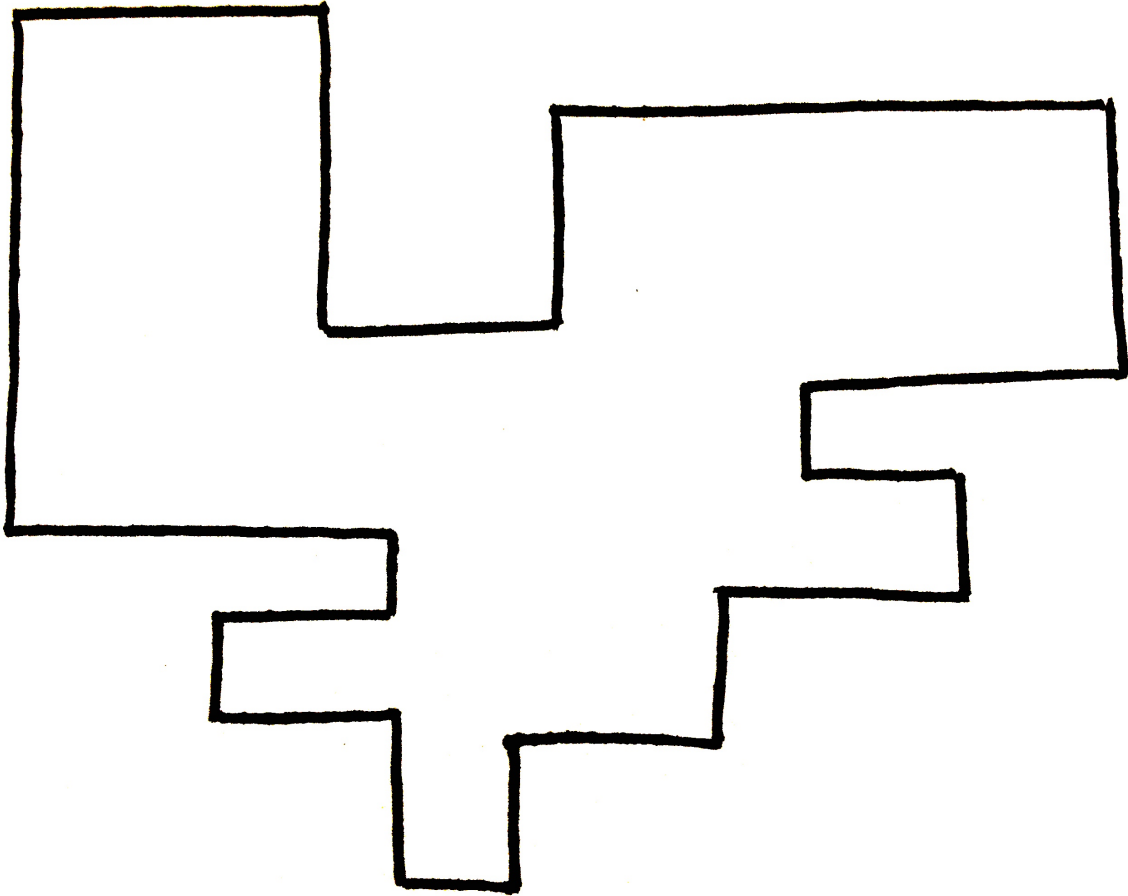
Show an example of where the guardsmen could be located. You may want to consider triangulating the gallery a few times to find the best placement. Show all of your work!

4. Explain in your own words why the triangulation and coloring technique works as a way to discover the number of guardsmen and where they should be located.

Part 2: Scoping out the Galileo Gallery

1. Cruella de Vil has stolen Elroy's new dalmatian puppy! She agrees to give it back only in exchange for a rare painting located in the Galileo Gallery which is guarded day and night. Elroy had visited that gallery before and knew it's floor plan (see below). However, he couldn't remember where all the guards were located. He also knew that the painting was located on the second floor next to the exit that goes out to the balcony.

   Suppose Elroy plans on gaining the painting through non-legal means. In order to retrieve it, Elroy would have to get past the guards on the first floor as well as the guards on the second floor. Lucky for Elroy, no guards guard the balcony of the gallery.

FLOOR 1

STAIRS

ENTRANCE & EXIT

FLOOR 2

STAIRS

PAINTING

BALCONY

EXIT (TO BALCONY)
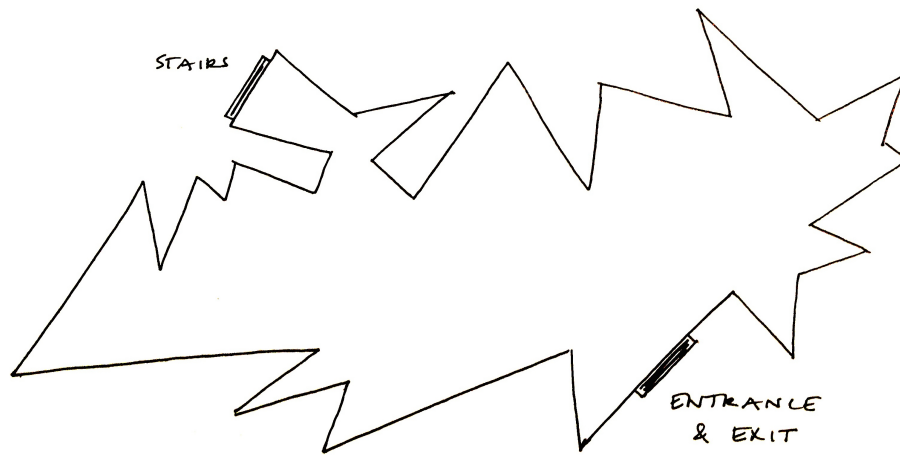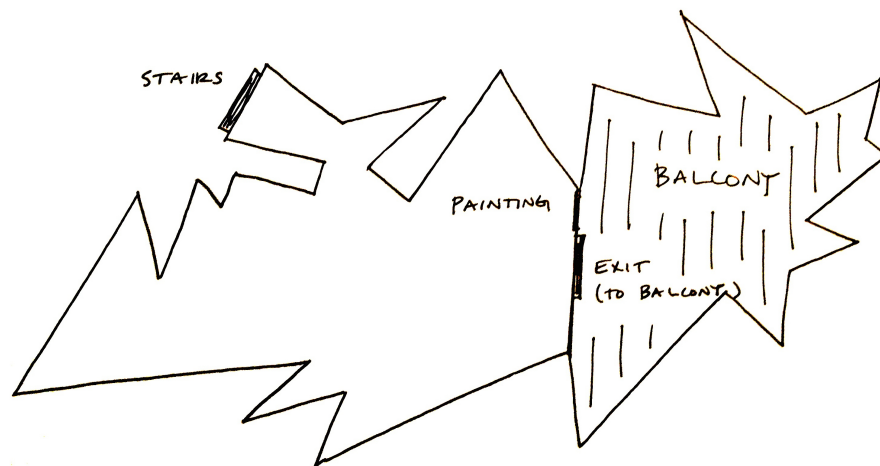
(a) Assuming that the gallery hires the least number of (vertex) guards needed to observe the entire gallery, where could Elroy expect the guards to be placed? Show your work!

(b) Will there be guarded-guards (each guard can be seen by at least one other guard)? Label the guards that are guarded-guards.

(c) If the gallery required that each guard could be seen by at least one other guard, how many guards are necessary to guard the entire gallery? Show an example of where the guards could be placed. Show your work!

(d) Instead of guarded-guards, suppose the guards of the gallery are restricted to a 180° field of vision (e.g. the guards can't see behind themselves). How many guards would be needed for the gallery? Show an example of where the guards could be placed. Show your work!

(e) Instead of the guarded-guards or guards with restricted vision, assume that the gallery allows the guards to be place anywhere inside the gallery (instead of the guards being restricted to the vertices as before). That is, the guards are point guards. Suppose the gallery hires the least number of (point) guards needed to observe the entire gallery. How many guards could Elroy expect at the gallery? Show an example of where the guards could be placed. Show your work!