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Project 2 - Voting Methods

Math 1030Q – Fall 2014

Professor Hohn

Show all of your work! Write neatly. No credit will be given to unsupported answers. Projects are due *at the beginning of class*. Any project not collected by the instructor at the beginning of class is considered late (and will receive 0 points on the project). No late projects will be accepted!

Part 1: Who wins the election

1. The homeowners association is deciding a new set of neighborhood standards for architecture, yard maintenance, etc. Four options have been proposed. The votes are:

| Number of voters | 8 | 9 | 11 | 7 | 7 | 5 |
|------------------------|---|---|----|---|---|---|
| 1 st choice | B | A | D | A | B | C |
| 2 nd choice | C | D | B | B | A | D |
| 3 rd choice | A | C | C | D | C | A |
| 4 th choice | D | B | A | C | D | B |

- (a) How many voters voted in this election?

- (b) How many votes are needed for a majority? A plurality?

(c) Find the winner under the plurality method.

(d) Find the winner under the Borda Count Method.

(e) Find the winner under the Instant Runoff Voting method.

2. Consider an election with 129 votes.

(a) If there are 4 candidates, what is the smallest number of votes that a plurality candidate could have? Explain your answer.

(b) If there are 8 candidates, what is the smallest number of votes that a plurality candidate could have? Explain your answer.

3. The Pareto criterion is another fairness criterion that states: *If every voter prefers choice A to choice B, then B should not be the winner.* Explain why plurality, instant runoff, and Borda count methods all satisfy the Pareto condition.

4. In a primary system, a first vote is held with multiple candidates. In some states, each political party has its own primary. In California, there is a “top two” primary, where all candidates are on the ballot and the top two candidates advance to the general election, regardless of party. Compare and contrast the “top two” primary with general election system to instant runoff voting, considering both differences in the methods, and practical differences like cost, campaigning, fairness, etc.

Part 2: The search for a leader

5. Elroy's determined to get his puppy back (via illegitimate means) by getting the painting from the gallery for Cruela Di Vil (we all know what Cruela does with dalmation puppies). He decides to call upon the help of several of his friends to help retrieve the painting. The team he assembles consists of Wyldstyle (aka Lucy), Megamind, Merida, Batman, Lisa Simpson, Kenny, and 9 of Gru's finest minions. They call themselves...the Group of Good (and sometimes evil)!



Who should lead the Group of Good? Although Elroy recruited everyone for his cause, he would like everyone to decide via voting who should lead the gallery heist. To let the group decide, Elroy has everyone fill out a preference ballot showing who he/she would like to be the leader of their group. Elroy tallies up the ballots to get the following preference schedule:

| | | | | |
|----------|----|----|----|----|
| | 2 | 3 | 2 | 1 |
| Choice 1 | W | MM | B | E |
| Choice 2 | M | W | MM | B |
| Choice 3 | LS | K | LS | M |
| Choice 4 | G | G | G | G |
| Choice 5 | B | LS | W | LS |
| Choice 6 | E | E | K | MM |
| Choice 7 | MM | M | M | K |
| Choice 8 | K | B | E | W |

Note that E is for Elroy, W is for Wyldstyle, MM is for Megamind, M is for Merida, B is for Batman, LS is for Lisa Simpson, K is for Kenny, and G is for Gru's finest minions. Gru's finest minions always vote for the same candidate and count as one vote.

- (a) Suppose Elroy wants Megamind to win (go De-Gun). What voting method (or methods) should he propose in order for Megamind to win (plurality, Instant Runoff, Borda Count, Vote-for-two, Approval voting (if each only approves of his/her top 4 candidates), etc)? Show how you came to that conclusion.

- (b) What voting method should Elroy employ to make everyone in the group feel like the election was “fair?” Which fairness criterion does the voting method satisfy? Why should he choose that method? Who would be the leader?

6. Now, the Group of Good would like to decide how they should infiltrate the gallery. The group wants to vote on different plans to sneak into the gallery; the plans are labelled $a - e$. While the group puts all of the plans together so everyone can see them, a tragedy occurs. Kenny gets a paper cut by Plan a ! The cut is too deep! Saddened by their colleague's sudden paper death, they decide that the leader should have more say in which plan is picked (in hopes to avoid more unnecessary death). Suppose the leader's vote (from the last question) counts as 5 votes. The following preference schedule is produced:

| | Leader | 2 | 2 | 1 | 1 |
|----------|--------|---|---|---|---|
| Choice 1 | e | c | a | b | b |
| Choice 2 | d | a | b | d | a |
| Choice 3 | c | d | e | c | d |
| Choice 4 | a | b | d | e | c |
| Choice 5 | b | e | c | a | e |

(a) If plurality voting is used, what plan should the group use? Does this plan have a majority?

(b) If Instant Runoff Voting is used, what plan should the group use? Show how you discovered your answer.

(c) If the Borda Count method is used, what plan should the group use? Show how the winner is computed.