

Project 4 - Graph Theory

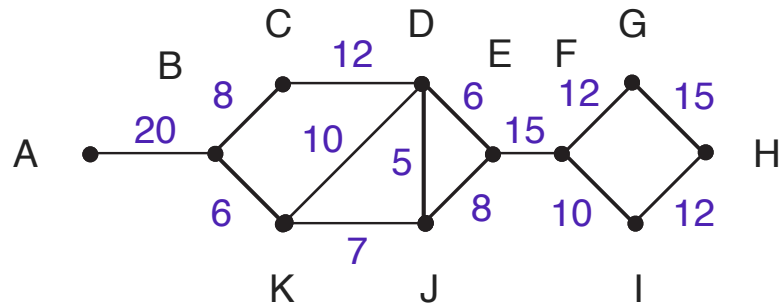
Math 1030Q – Fall 2014

Professor Hohn

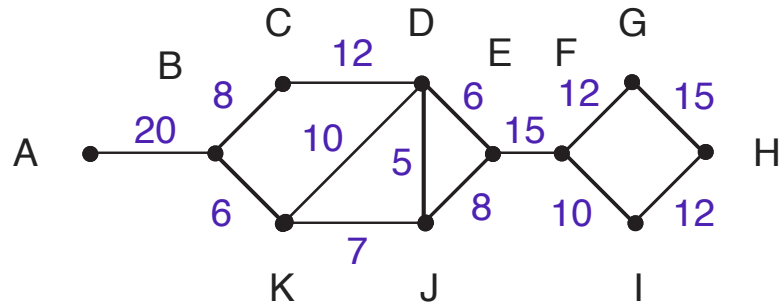
Show all of your work! Write neatly. No credit will be given to unsupported answers. Projects are due *at the beginning of class*. Any project not collected by the instructor at the beginning of class is considered late (and will receive 0 points on the project). No late projects will be accepted!

Part 1:

- Use Kruskal's algorithm to find a minimal spanning tree for the given weighted graph below. List the edges in the order they are selected, sketch the minimal spanning tree, and give the total weight of the minimal spanning tree.



2. Modify Kruskal's algorithm to find a maximal spanning tree for the given weighted graph below. List the edges in the order they are selected, sketch the maximal spanning tree, and give the total weight of the maximal spanning tree.



3. Another approximation algorithm for finding low-cost Hamiltonian circuits is called the Cheapest-link algorithm. The idea is that we create a Hamiltonian circuit by always picking the lowest-cost edge that does not add a third edge at any vertex or create a cycle (other than the one that completes the Hamiltonian circuit). Here is a description of the algorithm:
 1. In the beginning, all edges are acceptable and no edges have been selected.
 2. For the set of acceptable edges, select the edge of smallest weight. If there is a tie, select any of the edges with the smallest weight.
 3. If the selected edges do not form a Hamiltonian circuit, then determine the set of acceptable edges. Unacceptable edges are those that either share one vertex with two selected edges or that would close a circuit that is not a Hamiltonian circuit.
 4. If the selected edges form a Hamiltonian circuit, that circuit is your low-cost Hamiltonian circuit.

Use this algorithm for the following question.

A student has errands to run: renting a movie from the video store, filling the car up at the gas station, buying potato chips and popcorn at the grocery store to eat while watching the video, and purchasing a new pair of running shoes at the shoe store. The following table shows the driving time (in minutes) between each of these locations.

Location	Video Store	Gas Station	Grocery Store	Shoe Store	Home
Video Store	–	2	5	10	8
Gas Station	2	–	1	11	8.5
Grocery Store	5	1	–	9	7
Shoe Store	10	11	9	–	6
Home	8	8.5	7	6	–

- (a) Draw a complete weighted graph to represent this traveling-salesperson problem.

- (b) Use the Cheapest-link algorithm to approximate a route the student might take in order to complete these errands in the shortest time, assuming that he/she starts and ends in the same place. What is the shortest driving time obtained by this method?

4. A group of Hobbits plan to visit five cities in Middle Earth during Thanksgiving week on a Thanksgiving ME Tour. There are some paths that are available between most cities. The following table lists the distance in miles between five cities the group plans to visit. Note that there are some paths that are unavailable between some cities.

Table 1: Hypothetical distances between cities of Middle Earth

Cities	The Shire	Rivendell	Lothlorien	Minas Tirith	Isengard
The Shire	–	56	162	141	32
Rivendell	56	–	191	118	Path unavailable
Lothlorien	162	191	–	Path unavailable	135
Minas Tirith	141	118	Path unavailable	–	123
Isengard	32	Path unavailable	135	123	–

- (a) Draw a weighted graph to represent the Thanksgiving ME Tour using the information in the table.

- (b) Find three different routes from Rivendell to Minas Tirith and calculate the total distance for each.

5. In your own words, explain the difference between an Euler circuit and a Hamiltonian circuit. Draw a graph depicting each type. Give an example of each circuit being used in your life.

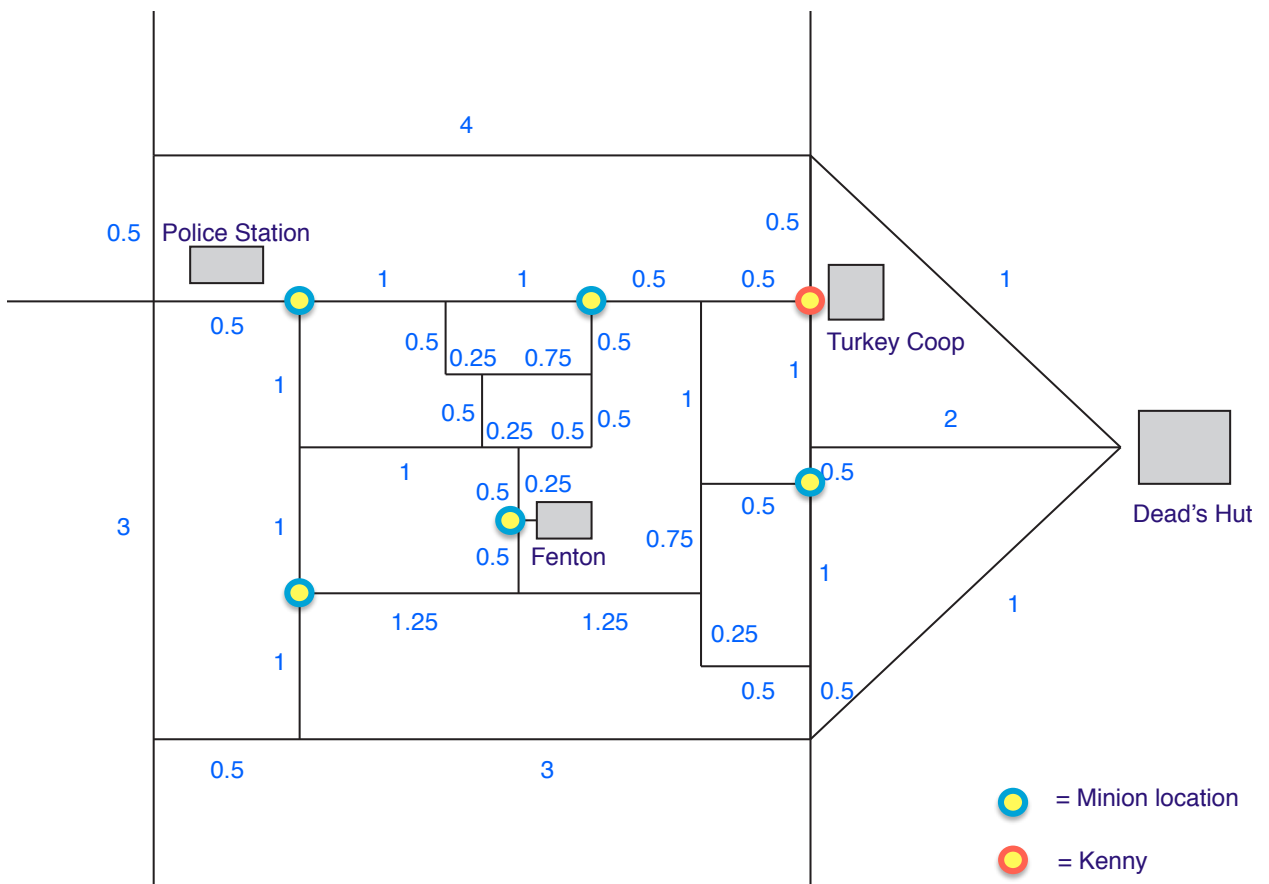
Part 2:

The Group of Good found out that the painting (the one they'd like to exchange for Elroy's Dalmatian puppy) is on a temporary exhibition at another art gallery called the Fenton located on the uCON campus. To the Group of Good's despair, the art gallery is in the middle of the uCON campus. The goal is to drive from their hideout at Dead's Hut to the Fenton as quickly as possible, retrieve the painting, and drive from the Fenton back to Dead's Hut.

6. Together, the Group of Good is planning their heist from Dead's Hut. In order to infiltrate the gallery and make a fast getaway, the Group of Good decides to position lookouts across the campus at specific locations.

Wyldestyle's master builder techniques will be used to build the getaway car, Megamind's De-gun will be used to make the painting pocket sized, and Kenny and the Minions will be the lookouts.

See map below. Weights are in miles.

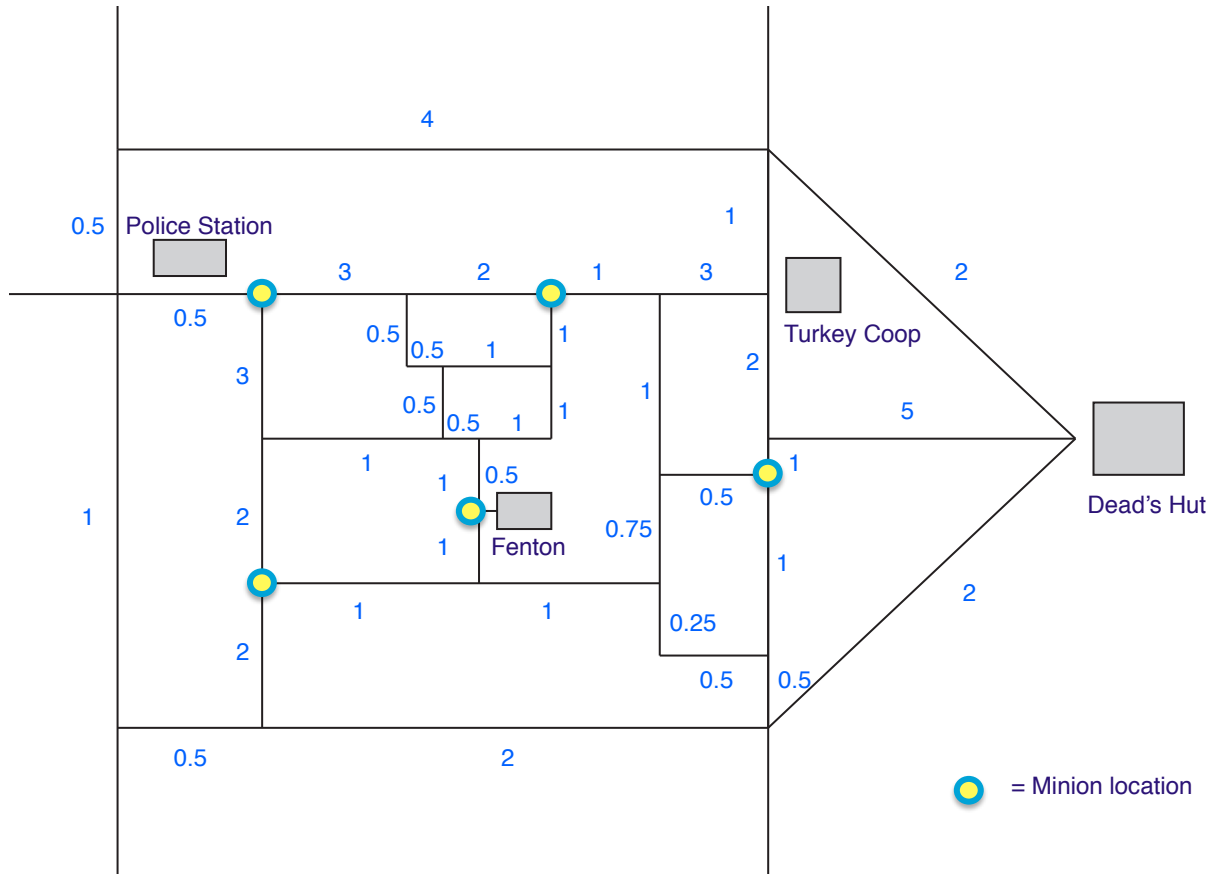


- (a) Find the quickest way (in miles) to get from Dead's Hut to the Fenton and back again. Draw your route and note its mileage. Remember to pick up all of the minions and Kenny *on the way back* from the Fenton.

- (b) The Group of Good is ready to go! The lookouts were placed earlier in the day, and the Group of Good has a plan to get to the Fenton and back to Dead's Hut. Kenny, one of the lookouts, places himself near the turkey farm located on the north end of campus. Kenny, interested in the turkey's flying abilities, opens the coup to let the turkeys out for observation. The turkeys fly out of the coup, claws scratching Kenny repeatedly. Oh, no! The largest turkey waddles out, flattening Kenny. The turkey sinks his claws into Kenny, and Kenny dies instantly. One of the minions, seeing the horrific events, signals the others that Kenny no longer needs to be picked up.

Now that Kenny is no longer in need of getting picked up, find the quickest way to get from the Fenton to Dead's Hut. Draw your route and note its mileage. Remember that you must pick up all of the minions.

(c) While the Group of Good was infiltrating the Fenton, classes let out at uCON. Students are crossing the streets and driving around frantically to get to and from class. Luckily, the Group of Good brought a map and estimated the time (in minutes) it would take on each road with traffic (see map below).



What is the fastest way (in minutes) to get back to Dead's Hut? Remember to pick up all of the minions. Draw your route and note its time.