

Score: \_\_\_\_\_

Name: \_\_\_\_\_

## Project 5 - Fair Division

Math 1030Q – Fall 2014

Professor Hohn

Show all of your work! Write neatly. No credit will be given to unsupported answers. Projects are due *at the beginning of class*. Any project not collected by the instructor at the beginning of class is considered late (and will receive 0 points on the project). No late projects will be accepted!

### Part 1:

One method of dividing something among three or more people is the Lone Divider Method. The method for  $N$  parties proceeds as follows:

1. The divider divides the item into  $N$  pieces, which the divider deems to be equal in value. Each piece is labeled.
2. Each of the choosers will separately list which pieces they deem to be their fair share. This is their declaration or bid.
3. The lists are examined. Two possibilities exist:
  - (a) If it is possible to give each party a piece they declared, then we give it to them. The divider gets the remaining piece.
  - (b) If two or more parties want the same piece and no other pieces, then the divider gets the uncontested piece. The rest of the pieces are combined and we repeat the entire process with the remaining parties. If only two parties are left, they can use the “I cut, you choose” method (also known as the Divider-chooser Method).

Here is an example.

Suppose we have three minions splitting a cake: Jerry, Stuart, and Dave. Each minion values the pieces differently: Piece 1 has a banana on it, Piece 2 has an apple, and Piece 3 has a potato. Their declarations for each piece is listed below:

	Piece 1	Piece 2	Piece 3
Jerry	40%	30%	30%
Stuart	40%	35%	25%
Dave (the Divider)	33.333%	33.333%	33.333%

Notice for the three minions, a fair share is a piece valued at  $33.\overline{33}\%$  or more. Looking at the table, we see that Jerry deems Piece 1 a fair share (40%), Stuart deems Piece 1 (40%) and Piece 2 (35%) as fair shares, and Dave deems them all fair shares. We could write it as follows:

Declarations:

Jerry: Piece 1

Stuart: Piece 1, Piece 2

Dave: Piece 1, Piece 2, Piece 3

Hence, we would distribute the pieces as follows: Piece 1 to Jerry, Piece 2 to Stuart, and Piece 3 to Dave. Notice that each minion has a piece that is considered a fair share to them.

1. After the fall of Sauron, the Hobbits, Elves, Dwarves, and the race of Men want to divide up the land of Mordor among themselves. They decide to use the Lone-divider method. The values of the four pieces of land in the eyes of the each group are shown below.

	Piece 1	Piece 2	Piece 3	Piece 4
Hobbits	21%	27%	32%	20%
Elves	27%	29%	22%	22%
Dwarves	23%	14%	41%	22%
The race of Men	25%	25%	25%	25%

- (a) Who was the divider?

**Solution:** The race of Men.

- (b) Supposing that each player is playing honestly, what will each player's declaration be?

**Solution:** Declarations:

Hobbits: Piece 2, Piece 3

Elves: Piece 1, Piece 2

Dwarves: Piece 3

The race of Men: Piece 1, Piece 2, Piece 3, Piece 4

- (c) Find the final division.

**Solution:** Dwarves receive Piece 3, Hobbits receive Piece 2, Elves receive Piece 1, and the race of Men get Piece 4.

## Part 2: Dead's Hut

After arriving safely in Dead's Hut, the members of Subgroup 1 (Elroy, Wyldstyle, and Megamind) dash to the back room to take a look at the painting that Megamind procured from the Fenton. Minions rush into the back room with water to rehydrate the painting, accidentally tripping and pouring water all over Megamind. The painting immediately begins growing, but for some reason, more paintings are sprouting all around Megamind. Megamind De-gunned 3 extra paintings! Subgroup 1 (plus the minions) decide to divide up the paintings fairly amongst themselves.

2. Suppose Subgroup 1 and the minions want to use the Sealed Bid Method. Their bids are listed below (in thousands of dollars).

	Painting 1 (Exchange for puppy)	Painting 2	Painting 3	Painting 4
Elroy	50	7	11	8
Wyldstyle	8	13	10	11
Megamind	3	1	5	4
Minions	7	6	3	8

- (a) What is Elroy's total valuation of the items? Wyldstyle's total valuation? Megamind's? The minions'?

**Solution:** Elroy

$$50 + 7 + 11 + 8 = 76$$

Wyldstyle

$$8 + 13 + 10 + 11 = 42$$

Megamind

$$3 + 1 + 5 + 4 = 13$$

Minions

$$7 + 6 + 3 + 8 = 24$$

- (b) What is Elroy's fair share in thousands of dollars? Wyldstyle's fair share? Megamind's? The minions'?

**Solution:** Elroy

$$76/4 = 19$$

Wyldstyle

$$42/4 = 10.5$$

Megamind

$$13/4 = 3.25$$

Minions

$$24/4 = 6$$

(c) Find the final allocation.

**Solution:** Elroy:

Elroy would get Painting 1 and Painting 3 because he is the highest bidder for each painting. The paintings in total would be  $\$50 + 11 = \$61$  thousand dollars. His fair share was  $\$19$  thousand. Then,  $\$61 - 19 = \$42$  thousand goes into the holding pile.

Wyldstyle:

Wyldstyle would get Painting 2 and Painting 4 because she is the highest bidder for each painting. The total worth of the paintings she received in her eyes was  $\$13 + 11 = \$24$  thousand. Her fair share was  $\$10.5$  thousand. Then,  $\$24 - 10.5 = \$13.5$  thousand goes into the holding pile.

Megamind and minions:

Megamind and the minions would not receive any paintings.

Holding pile distribution:

The holding pile would have  $\$42 + 13.5 = \$55.5$  thousand. We give Megamind and the minions their fair share from the pile.

Megamind gets  $\$3.25$  thousand, and the minions get  $\$6$  thousand. We take that from the holding pile to get  $\$55.5 - 9.25 = 46.25$  thousand. We divide that by 4 and distribute to everyone.

$$\$46.25/4 = \$11.5625$$

Distribution:

In the end, Elroy would get Painting 1, Painting 3, and give  $\$42 - 11.5625 = \$30.4375$  thousand dollars to the others. Wyldstyle would get Painting 2, Painting 4, and give  $\$13.5 - 11.5625 = \$1.9375$  thousand dollars. Megamind would get  $\$3.25 + 11.5625 = \$14.8125$  thousand dollars, and the minions would get  $\$6 + 11.5625 = \$17.5625$  thousand dollars.

Meanwhile, the members of Subgroup 2 (Merida, Batman, Lisa, and Kenny) head to the refrigerator in Dead's Hut. A large cake with a picture of the Group of Good has been made to celebrate their return from the Fenton. Subgroup 2, wondering why it is taking so long to get the painting rehydrated, decide to delve into the cake themselves. They carefully put the candles on the cake and prepare to blow them out together. Oh, no! The candles near Kenny are getting out of control. The flames leap onto Kenny! Kenny burns to death! The rest of Subgroup 2 blows out the conflagration on the cake. Now, the cake will be divided up among the three of them.

3. Lisa volunteers to be the divider for the Lone-divider method. She divides the cake into three pieces: Piece 1 has a picture of the Group of Good, Piece 2 has the most frosting, and Piece 3 has more cake than frosting. Merida and Batman write down their declarations for the different pieces of cake. See below.

	Piece 1	Piece 2	Piece 3
Merida	25%	40%	35%
Batman	40%	35%	25%
Lisa (the Divider)	33.333%	33.333%	33.333%

- (a) Supposing that each player is playing honestly, what will each player's declaration be?

**Solution:** Declarations:

Merida: Piece 2, Piece 3

Batman: Piece 1, Piece 2

Lisa: Piece 1, Piece 2, Piece 3

- (b) Find the final division. If there is more than one way to divide up the cake fairly, write down all of the possibilities.

**Solution:** Three possibilities:

- Batman receives Piece 1. Merida gets Piece 2. Lisa gets Piece 3.
- Batman receives Piece 1. Merida gets Piece 3. Lisa gets Piece 2.
- Batman receives Piece 2. Merida gets Piece 3. Lisa gets Piece 1.