Calculating Logarithms

Logarithmic functions are the inverse of exponential functions. Look at an example. Let $f(x) = 2^x$. Since f is a 1-1 function, it has an inverse. There's no nice formula for the inverse, so we give it a name and don't worry about a formula: $f^{-1}(x) = \log_2 x$. Since $f(3) = 8$, then $f^{-1}(8) = 3$, or in other words, $\log_2 8 = 3$. Your big key to computing logarithms is the following relationship:

$$y = \log_a x \text{ if and only if } x = a^y.$$

We can do a few more. Wondering what $\log_3 \frac{1}{3}$ is? Well, if $\log_3 \frac{1}{3} = ?$, then $3^? = \frac{1}{3}$. The ? must be -1. Wondering what $\log_5 1$ is? if $\log_5 1 = ?$, then $5^? = 1$. Therefore, $\log_5 1 = 0$. Your turn.

Find the value of each logarithm.

(1) $\log_2 16$

(2) $\log_5 125$

(3) $\log_7 7$

(4) $\log_{36} 6$

(5) $\log_4 \frac{1}{16}$

(6) $\log_9 9^{27}$

(7) $\log_\pi \pi$

(8) $\log_{10} 10000$

Let's make it a little trickier. Suppose we want to know what $\log_9 27$ is. If $\log_9 27 = ?$, then $9^? = 27$. You probably don't immediately see how to write 27 as a power of 9, but we can still simplify further because of something 9 and 27 have in common. Note that 9 and 27 are both powers of 3, so we can rewrite this equation as $(3^2)^? = 3^3$, or $3^{2?} = 3^3$. Then $2? = 3$, and $? = \frac{3}{2}$. Here are a few more examples where you'll need to get the bases of the exponential equations to match up:

(9) $\log_4 \frac{1}{8}$

(10) $\log_4 32$

(11) $\log_{16} 8$