Rules of Logarithms

Just like there are rules of exponents, there are rules of logarithms. They should have already been explained to you elsewhere, so here we'll just list them.

(1) $\log_r(AB) = \log_r A + \log_r B$.
(2) $\log_r \frac{A}{B} = \log_r A - \log_r B$.
(3) $\log_r(A^B) = B \log_r A$.
(4) $\log_r r^A = A$.
(5) $r^{\log_r A} = A$.

These rules can be used to simplify expressions.

(1) Use the rules of logarithms to write the following expressions as logarithms of one quantity with coefficient 1.

   (a) $\frac{1}{2} \ln x + \ln 5$

   (b) $\log_2 x + 4 \log_2(x+1) - \frac{1}{3} \log_2(x-1)$

   (c) $5 \ln x + 2 \ln 3 - 3 \ln \left(\frac{1}{y}\right)$

(2) Use the rules of logarithms to expand the following expressions so that there are no logarithms of products, quotients, or powers.

   (a) $\ln \sqrt[3]{x^3 y}$

   (b) $\log_{10} \frac{10}{4x^2}$

   (c) $\ln \left(\frac{x\sqrt{y}}{(1+x)^3}\right)$

(3) Suppose $\ln x = 2$, $\ln y = 3$ and $\ln z = 6$. Evaluate the following.

   (a) $\ln(xyz)$

   (b) $\ln(x^2 y)$

   (c) $\ln \left(\frac{x^3}{\sqrt{z}}\right)$