Homework Solutions

Math 1060Q: Precalculus

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Absolute Value Worksheet

Question 9

Find all possible values for x where $|x + 3| - |x - 2| < 1$.

Solution. Notice that the definition of the absolute value gives us:

$$|x + 3| = \begin{cases} x + 3 & \text{if } x + 3 \geq 0 \\ -(x + 3) & \text{if } x + 3 < 0. \end{cases}$$

Rewritten,

$$|x + 3| = \begin{cases} x + 3 & \text{if } x \geq -3 \\ -(x + 3) & \text{if } x < -3. \end{cases}$$

And,

$$|x - 2| = \begin{cases} x - 2 & \text{if } x - 2 \geq 0 \\ -(x - 2) & \text{if } x - 2 < 0. \end{cases}$$

Rewritten,

$$|x - 2| = \begin{cases} x - 2 & \text{if } x \geq 2 \\ -(x - 2) & \text{if } x < 2. \end{cases}$$

This means we have four cases:

1. $x \geq -3$ and $x \geq 2$ (that is, $x \geq 2$)

2. $x < -3$ and $x \geq 2$ (no such x exists, throw this case out)

3. $x \geq -3$ and $x < 2$ (that is, $x \in [-3, 2)$)

4. $x < -3$ and $x < 2$ (that is, $x < -3$)

You can think of a number line divided into sections: $(-\infty, -3)$, $[-3, 2)$, and $[2, \infty)$.

Now, let's check each case.

Case 1: If $x \in [2, \infty)$, then $|x + 3| = x + 3$ and $|x - 2| = x - 2$. Let's plug that into our equation.

$$x + 3 - (x - 2) < 1$$
$$5 < 1$$

This is not true for any $x \in (2, \infty)$. Hence, $x \notin (2, \infty)$.

Case 3: If $x \in [-3, 2)$, then $|x + 3| = x + 3$ and $|x - 2| = -(x - 2)$. Let's plug that into our equation.

$$x + 3 + (x - 2) < 1$$
$$2x + 1 < 1$$
$$2x < 0$$
$$x < 0$$

Hence, for Case 3, if $x \in [-3, 2)$, then x must also be less than 0. Hence, $x \in [-3, 0)$.

Case 4: If $x < -3$, then $|x + 3| = -(x + 3)$ and $|x - 2| = -(x - 2)$. Let's plug that into our equation.

$$-(x + 3) + (x - 2) < 1$$
$$-3 - 2 < 1$$
$$-5 < 1$$

For Case 4, all $x \in (-\infty, -3)$ make our equation true. Hence, $x \in (-\infty, -3)$.

Therefore, $|x + 3| - |x - 2| < 1$ when $x \in (\infty, -3)$ or $x \in [-3, 2)$. That is, $x \in (-\infty, 0)$. □