

Solving Trig Equations III

Find all solutions x in the interval $[0, 2\pi)$.

1. $\cos(2x) = -\frac{\sqrt{2}}{2}$

2. $\sec(3x) = -1$

3. $\sin\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

4. $\tan^2(2x) = 3$

Trigonometric Identities

- $\sin^2 x + \cos^2 x = 1$
- $\tan^2 x + 1 = \sec^2 x$
- $\cot^2 x + 1 = \csc^2 x$
- $\sin(x_1 \pm x_2) = \sin x_1 \cos x_2 \pm \cos x_1 \sin x_2$
- $\cos(x_1 \pm x_2) = \cos x_1 \cos x_2 \mp \sin x_1 \sin x_2$
- $\sin(2x) = 2 \sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$
- $\sin^2\left(\frac{x}{2}\right) = \frac{1 - \cos x}{2}$
- $\cos^2\left(\frac{x}{2}\right) = \frac{1 + \cos x}{2}$
- $\sin x_1 \cos x_2 = \frac{1}{2} (\sin(x_1 - x_2) + \sin(x_1 + x_2))$
- $\cos x_1 \cos x_2 = \frac{1}{2} (\cos(x_1 - x_2) + \cos(x_1 + x_2))$
- $\sin x_1 \sin x_2 = \frac{1}{2} (\cos(x_1 - x_2) - \cos(x_1 + x_2))$

Using the trigonometric identities above, find all solutions x in $[0, 2\pi)$.

1. $\sin(2x) = \cos x$

2. $2 \cot^2 x = 3 \csc x$

3. $\cos(4x) = \cos(2x)$

4. $2 \cos^2 \left(\frac{x}{2} \right) + \sin^2 x = 0$