Solving Trig Equations, Part II

By now you can solve some basic trig equations – typically, equations that involve only one trig function and perhaps some algebra first to get it into the form "trig function = a number," at which point you can use the unit circle to find your solutions. However, trig equations can get more complicated. They may involve multiple trig functions, or more complicated algebra. Here's a simple example that involves both:

$$\sin x \cos x + \frac{\cos x}{2} = 0$$

Here we're dealing with both sine and cosine in the same equation. We still want to reduce it to the form "trig function = a number," though, and our key to that is factoring. We can factor a cosine out of both terms in this equation:

$$\cos x(\sin x + \frac{1}{2}) = 0$$

And then break this equation into two parts. Since it's of the form $AB = 0$, we can split it up into $A = 0$ or $B = 0$:

$$\cos x = 0 \qquad \text{or} \qquad \sin x + \frac{1}{2} = 0$$

Then each of these is easy to solve. Let's restrict ourselves to the interval $[0, 2\pi]$ this time. If $\cos x = 0$, then $x = \frac{\pi}{2}, \frac{3\pi}{2}$, and if $\sin x = -\frac{1}{2}$, then $x = \frac{7\pi}{6}, \frac{11\pi}{6}$, so there are four possible solutions to this equation: $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

Sometimes, you may need to rewrite trig functions in terms of others:

$$\cos x = \cot x$$
$$\cos x - \cot x = 0$$
$$\cos x - \frac{\cos x}{\sin x} = 0$$
$$\cos x(1 - \frac{1}{\sin x}) = 0$$
$$\cos x = 0 \qquad \text{or} \qquad 1 - \frac{1}{\sin x} = 0$$

These reduce to $\cos x = 0$ or $\sin x = 1$, which gives us possible solutions $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

And sometimes, you may need to do other kinds of factoring:

$$\sin^2 x + \sin x - 2 = 0$$
$$(\sin x + 2)(\sin x - 1) = 0$$

$$\sin x + 2 = 0 \qquad \text{or} \qquad \sin x - 1 = 0$$

The equation $\sin x = -2$ has no solutions, since $\sin x$ is always a value between -1 and 1. The equation $\sin x = 1$ has solution $\frac{\pi}{2}$, and that is the only solution to this equation.

Now you try. Find all solutions to the following equations in the interval $[0, 2\pi]$.

(1) $2 \sin t \cos t = \sin t$

(2) $2 \sin^2 t + \sqrt{3} \sin t = 0$

(3) $2\cos^2 t + \cos t - 1 = 0$

(4) $\sin t + \tan t = 0$

(5) $\sin t = \cos t$

(6) $2\sin^2 t - 3\sin t + 1 = 0$

(7) $\tan t \sec t + \sqrt{2}\tan t = 0$