The Unit Circle

The unit circle is a circle of radius one, centered right at the origin. It's a great way for us to represent and measure angles, and is very useful for trigonometry. Most students are used to measuring angles in terms of degrees, so we'll start there. On the unit circle, we measure angles starting at the point $(1, 0)$, so that's 0 degrees. Angles go up as you travel counterclockwise around the circle (why counterclockwise? Just because someone said so a long time ago..). You travel around to 180 degrees, on the opposite side of the circle (at the point $(-1, 0)$) and then keep going until you get to 360 degrees and you're back where you started. You could keep going, if you like – to 450 degrees, 540 degrees, around to 720 degrees, and you're back where you started again.

If you travel in the opposite direction (clockwise), that's recorded as a negative angle. So, if you turn, say, 270 degrees, then you would end up facing the same direction as if you turned negative 90 degrees. Even though you end up in the same place, the angles turned *are* different. (After all, if someone asked you to turn around 10 times (3600 degrees) and run a race or not turn at all (0 degrees) and then take 5 steps, you'd do much better by not turning at all...even though you would start out pointing the same direction in either case.)

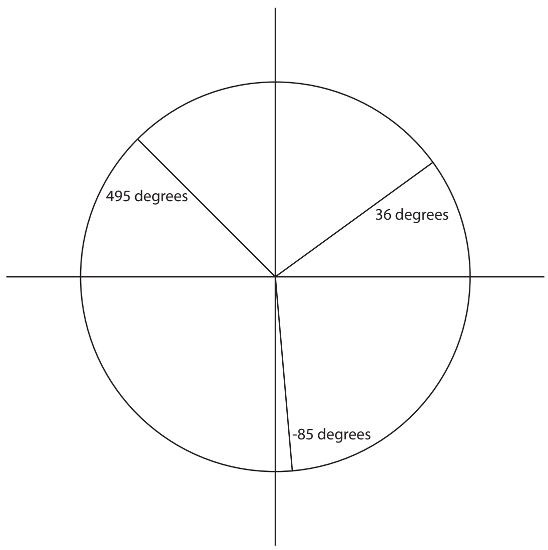

Figure 1.

In mathematics, rather than use degrees, we measure angles in radians (the difference between degrees and radians is like the difference between inches and centimeters, just two ways of measuring the same thing). In radians, one loop around the circle is 2π radians. You can use that to figure out the correspondence. If 360 degrees $= 2\pi$ radians, then 180 degrees $= \pi$ radians, and 90 degrees $= \frac{\pi}{2}$ radians, and so on. There's a formula you can look up if you like.

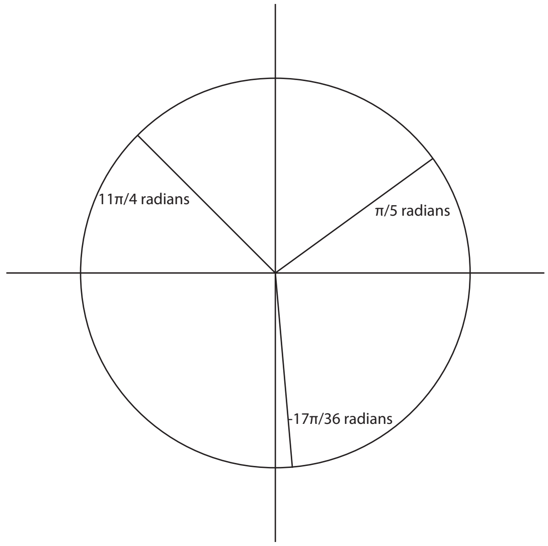

FIGURE 2.

(1) Plot the approximate location of the following angles on the unit circle.
  (a) $-405°$
  (b) $\frac{7\pi}{6}$ radians
  (c) 53π radians
  (d) $2\pi°$
  (e) 15 radians
  (f) $160°$
  (g) $-21\pi/5$ radians
  (h) 0.4 radians
  (i) $680°$
  (j) $-\frac{15\pi}{4}$ radians

There are some angles that come up often enough that we like to memorize them. We like to know where they are on the unit circle, *and* the (x, y) coordinates of the points on the unit circle where they are. Basically, the angles we memorize are multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$. Since $\frac{\pi}{3} = 2 \cdot \frac{\pi}{6}$, these include multiples of $\frac{\pi}{3}$ (and similarly, multiples of $\frac{\pi}{2}$). The following picture includes all of the angles on the unit circle you should know. It may seem like a lot at first, but there are patterns you can find. If you connect each point on the circle to the x-axis, you get a triangle, and in this picture, there are only "long skinny" triangles that have sides of length $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$, and "short fat" triangles that have sides of length $\frac{\sqrt{2}}{2}$ and $\frac{\sqrt{2}}{2}$.

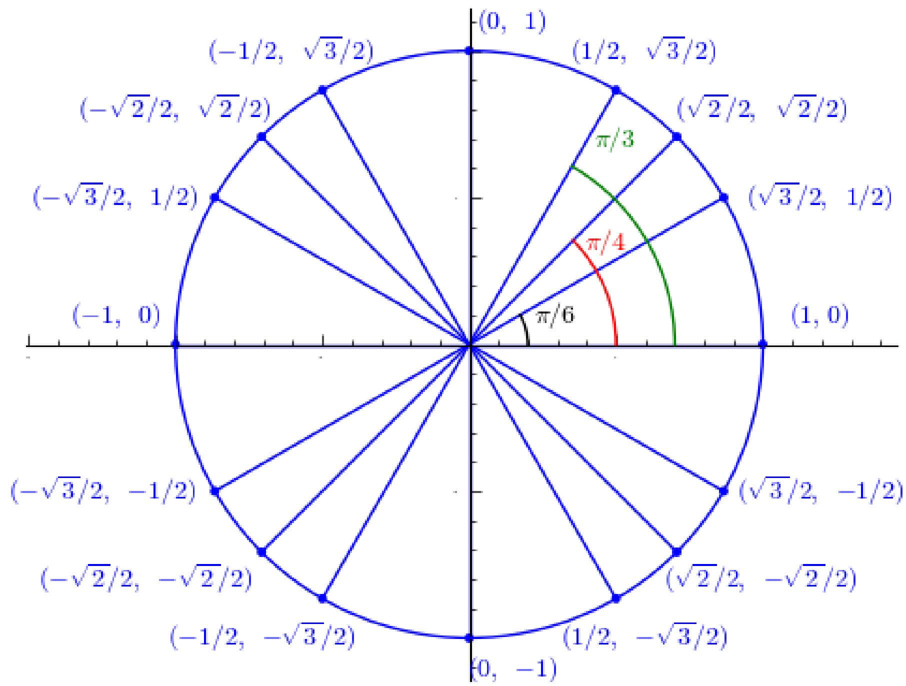

FIGURE 3.

(2) Come up with a way to memorize all these angles. Memorize them, even if just until the day after the final exam.