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Name:

## Review Worksheet - Chapter 15 + Lagrange multipliers

## Math 2110Q - Fall 2014

## Professor Hohn

You must show all of your work to receive full credit! Answer (in no particular order):

$$
176, \frac{8}{15}, e-2, \frac{1}{4},-\ln (2), \frac{1}{3}\left(2^{3 / 2}-1\right), \frac{\pi}{6}, \frac{13}{24}, \frac{64 \pi}{9}, \sqrt{2}, 16, \frac{1}{4}(e-1),-\sqrt{2}, 12 \pi
$$

1. Calculate the integral

$$
\int_{0}^{1} \int_{0}^{1} y e^{x y} d x d y
$$

2. Calculate the integral

$$
\int_{0}^{1} \int_{0}^{y} \int_{x}^{1} 6 x y z d z d x d y
$$

3. Write $\iint_{R} f(x, y) d A$ as an iterated integral where $R$ is the region described below.


Check your answer by letting $f(x, y)=1$.
4. Set up an integral

$$
\iint_{D} x d A
$$

where $D$ is the region in the first quadrant that lies between $x^{2}+y^{2}=1$ and $x^{2}+y^{2}=2$.
5. Set up an integral

$$
\iiint_{E} z d V
$$

where $E$ is the region bounded by $y=0, z=0, x+y=2$ and $y^{2}+z^{2}=1$ in the first octant.
6. Set up an integral to find the volume of the solid bounded by $x^{2}+y^{2}=4, z=0$, and $y+z=3$.
7. Set up an integral to find the volume of the solid under the paraboloid $z=x^{2}+4 y^{2}$ and about the rectangle $R=[0,2] \times[1,4]$.
8. Set up an integral to find the volume of the solid above the paraboloid $z=x^{2}+y^{2}$ and below the half cone $z=\sqrt{x^{2}+y^{2}}$.
9. Convert the following integral into an integral with spherical coordinates.

$$
\int_{-2}^{2} \int_{0}^{\sqrt{4-y^{2}}} \int_{-\sqrt{4-x^{2}-y^{2}}}^{\sqrt{4-x^{2}-y^{2}}} y^{2} \sqrt{x^{2}+y^{2}+z^{2}} d z d x d y
$$

10. Rewrite the integral

$$
\int_{-1}^{1} \int_{x^{2}}^{1} \int_{0}^{1-y} f(x, y, z) d z d y d x
$$

as an iterated integral in the order $d x d y d z$. Check your answer by integrating using the function $f(x, y, z)=1$.
11. Calculate the integral below by first reversing the order of integration.

$$
\int_{0}^{1} \int_{\sqrt{y}}^{1} \frac{y e^{x^{2}}}{x^{3}} d x d y
$$

12. Use Lagrange multipliers to find the maximum and minimum values of $f(x, y)=\frac{1}{x}+\frac{1}{y}$, subject to the constraint $\frac{1}{x^{2}}+\frac{1}{y^{2}}=1$.
13. Bonus: Use the transformation $u=x-y, v=x+y$ to evaluate

$$
\iint_{R} \frac{x-y}{x+y} d A
$$

where $R$ is the square with vertices $(0,2),(1,1),(2,2)$, and $(1,3)$.

