Ch. 15 Review

MATH 2110Q – Fall 2015
Professor Hohn

You must show all of your work to receive full credit!
Solutions (in no particular order):

$$176, \frac{8}{15}, e-2, \frac{1}{4}, -\ln(2), \frac{1}{3}(2^{3/2}-1), \frac{\pi}{6}, \frac{13}{24}, \frac{64\pi}{9}, 16, \frac{1}{4}(e-1), 12\pi, 0, \pi/12, 9\pi(\sqrt{2}-1)$$

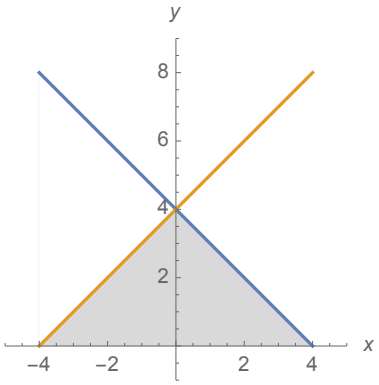
1. Calculate the integral

$$\int_0^1 \int_0^1 ye^{xy}\,dx\,dy.$$

2. Calculate the integral

$$\int_0^1 \int_0^y \int_x^1 6xyz \, dz \, dx \, dy.$$

3. Write $\iint_R f(x,y)\,dA$ as an iterated integral where R is the region described below.



Check your answer by letting $f(x,y) = 1$.

4. Convert the following into an integral in cylindrical coordinates.

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\,dz\,dx\,dy$$

(Check your answer by integrating and using the function $f(x,y,z)=1$ instead of the function $f(x,y,z)=xyz$)

5. Convert the following into an integral in spherical coordinates.

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2+y^2+z^2)\, dz\, dx\, dy$$

(Check your answer by integrating and using the function $f(x, y, z) = 1$ instead of $f(x, y, z) = x^2 + y^2 + z^2$)

6. Set up an integral

$$\iint_D x \, dA$$

where D is the region in the first quadrant that lies between $x^2 + y^2 = 1$ and $x^2 + y^2 = 2$.

7. Set up an integral

$$\iiint_E z \, dV$$

where E is the region bounded by $y = 0, z = 0, x + y = 2$ and $y^2 + z^2 = 1$ in the first octant.

8. Set up an integral to find the volume of the solid bounded by $x^2 + y^2 = 4$, $z = 0$, and $y + z = 3$.

9. Set up an integral to find the volume of the solid under the paraboloid $z = x^2 + 4y^2$ and about the rectangle $R = [0, 2] \times [1, 4]$.

10. Set up an integral to find the volume of the solid above the paraboloid $z = x^2 + y^2$ and below the half cone $z = \sqrt{x^2 + y^2}$.

11. Convert the following integral into an integral with spherical coordinates.

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy$$

12. Rewrite the integral

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} f(x,y,z)\,dz\,dy\,dx$$

as an iterated integral in the order $dx\,dy\,dz$. Check your answer by integrating using the function $f(x,y,z) = 1$.

13. Calculate the integral below by first reversing the order of integration.

$$\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}\, dx\, dy$$

14. Bonus: Use the transformation $u = x - y$, $v = x + y$ to evaluate

$$\iint_R \frac{x-y}{x+y}\, dA$$

where R is the square with vertices $(0, 2), (1, 1), (2, 2)$, and $(1, 3)$.