Graded by: _____ Score: _____ ID: _____

WORKSHEET 6 - DUE 10/12

MATH 2110Q – Fall 2015
Professor Hohn

You must show all of your work for full credit. Please circle/box your answers or write a brief sentence indicating your answer.

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1. Find the equation of the tangent plane to the surface at the given point.
   (a) $z = 3y^2 - 2x^2 + x,\ (2, -1, -3)$

   (b) $z = x\sin(x + y),\ (-1, 1, 0)$

2. The temperature at point (x, y, z) is given by

$$T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$$

where T is measured in $^\circ C$ and x, y, z in meters.

(a) Find the rate of change of temperature at the point $P(2, -1, 1)$ in the direction toward the point $Q(3, -3, 3)$.

(b) In which direction does the temperature increase the fastest at P?

(c) Find the maximum rate of increase at P.

3. Let $g(x, y) = x^2 + y^2 - 4x$.

   (a) Find the gradient vector $\nabla g(1, 2)$ and use it to find the tangent line to the level curve $g(x, y) = 1$ at the point $(1, 2)$.

   (b) Sketch the level curve, the tangent line, and the gradient vector. Label each one clearly.

4. A function is called homogeneous of degree n if it satisfies the equation $f(tx, ty) = t^n f(x, y)$ for all t, where n is a positive integer and f has continuous second-order partial derivatives.

   (a) Verify that $f(x, y) = x^2y + 2xy^2 + 5y^3$ is homogeneous of degree 3.

   (b) Show that f satisfies the equation

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3f(x, y).$$