Graded by: _____ Score: _____ ID: _____

Worksheet 1 - Due 9/9

MATH 2110Q – Fall 2015
Professor Hohn

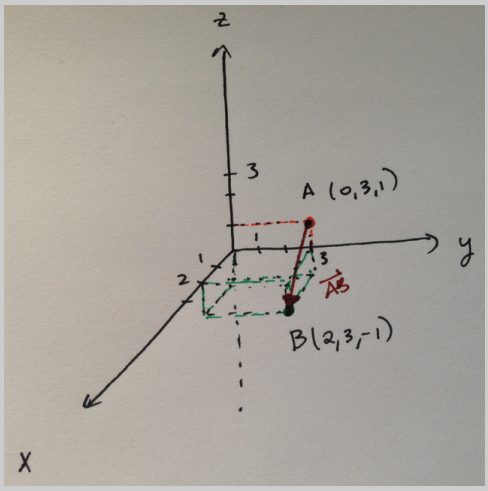
You must show all of your work to receive full credit!

1. Find a vector $\vec{a}$ with representation given by the directed line segment $\overrightarrow{AB}$ where $A(0,3,1)$ and $B(2,3,-1)$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

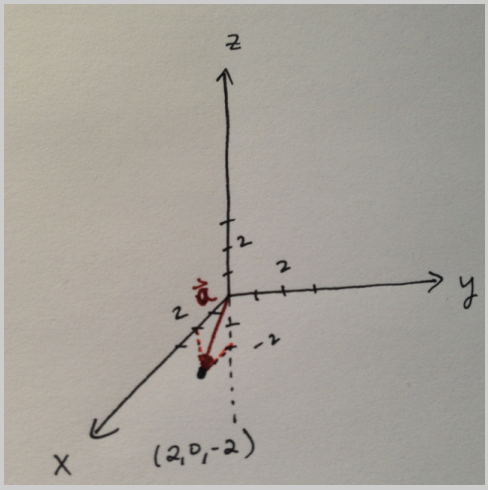
Solution: Let $\vec{v}$ be the vector with representation given by the directed line segment $\overrightarrow{AB}$ where $A(0,3,1)$ and $B(2,3,-1)$. Then,

$$\vec{a} = \langle 2-0, 3-3, -1-1 \rangle = \langle 2, 0, -2 \rangle$$

The drawing of $\overrightarrow{AB}$ is



The drawing of $\vec{a}$ is



2. Let $\vec{a} = 2\hat{x} - 4\hat{y} + 4\hat{z}$ and $\vec{b} = 2\hat{y} - \hat{z}$. Compute

(a) $\vec{a} + \vec{b}$

Solution:

$$\vec{a} + \vec{b} = \langle 2 + 0, -4 + 2, 4 - 1 \rangle = \langle 2, -2, 3 \rangle$$

(b) $2\vec{a} + 3\vec{b}$

Solution:

$$2\vec{a} = \langle 4, -8, 8 \rangle$$

$$3\vec{b} = \langle 0, 6, -3 \rangle$$

Then,

$$2\vec{a} + 3\vec{b} = \langle 4 + 0, -8 + 6, 8 - 3 \rangle = \langle 4, -2, 5 \rangle$$

(c) $\|\vec{a}\|$

Solution:

$$\|\vec{a}\| = \sqrt{2^2 + (-4)^2 + 4^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$$

(d) $\left\|\vec{a} - \vec{b}\right\|$

Solution:

$$\vec{a} - \vec{b} = \langle 2 - 0, -4 - 2, 4 - (-1) \rangle = \langle 2, -6, 5 \rangle$$

Then,

$$\left\|\vec{a} - \vec{b}\right\| = \sqrt{2^2 + (-6)^2 + 5^2} = \sqrt{4 + 36 + 25} = \sqrt{65}$$

3. Let $\vec{v} = \langle -4, 2, 2 \rangle$.

(a) Find the unit vector that has the same direction as $\vec{v}$.

Solution: Let $\hat{u}$ be the unit vector in the same direction as $\vec{v}$. Then,

$$\hat{u} = \frac{\vec{v}}{\|\vec{v}\|}$$

We find $\|\vec{v}\|$:

$$\|\vec{v}\| = \sqrt{(-4)^2 + 2^2 + 2^2} = \sqrt{16 + 4 + 4} = \sqrt{24}$$

Then,

$$\hat{u} = \langle \frac{-4}{\sqrt{24}}, \frac{2}{\sqrt{24}}, \frac{2}{\sqrt{24}} \rangle$$

(b) Find the vector that has the same direction as $\vec{v}$, but has length 6.

Solution: We know from part (a) that $\hat{u}$ is a vector in the direction of $\vec{v}$ of length 1 (since $\vec{u}$ is a unit vector). Thus, we can find a vector $\vec{w}$ that has the same direction as $\vec{v}$ but with length 6 by multiplying $\hat{u}$ by 6. Then,

$$\vec{w} = 6\hat{u} = \langle \frac{-24}{\sqrt{24}}, \frac{12}{\sqrt{24}}, \frac{12}{\sqrt{24}} \rangle$$

4. Application Question

A quarterback throws a football with angle of elevation 40° and speed 60 ft/s. Find the horizontal and vertical components of the velocity vector.

Solution: We know a couple things about the velocity vector $\vec{a}$: 1) the magnitude (60 ft/s), and 2) the angle (40°). Then, the velocity vector $\vec{a}$ would look like:

$$\vec{a} = \langle 60\cos(40^\circ), 60\sin(40^\circ) \rangle$$