Graded by: ________________ Score: ______ ID: __________________

WORKSHEET 5 - DUE 10/5

MATH 2110Q – Fall 2015
Professor Hohn

You must show all of your work to receive full credit!

1. Compute $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$, and $\frac{\partial^2 f}{\partial y^2}$ for the following functions at the indicated point.

   (a) $f(x, y) = xy; (1, 1)$

   Solution:

$$f_x(x, y) = y \implies f_x(1, 1) = 1$$
$$f_y(x, y) = x \implies f_y(1, 1) = 1$$
$$f_{xy} = (f_x)_y = 1 \implies f_{xy}(1, 1) = 1$$
$$f_{yx} = (f_y)_x = 1 \implies f_{yx}(1, 1) = 1$$
$$f_{xx} = 0 \implies f_{xx}(1, 1) = 0$$
$$f_{yy} = 0 \implies f_{yy}(1, 1) = 0$$

   (b) $f(x, y) = \frac{x}{y}; (1, 1)$

   Solution:

$$f_x(x, y) = \frac{1}{y} \implies f_x(1, 1) = 1$$
$$f_y(x, y) = \frac{-x}{y^2} \implies f_y(1, 1) = -1$$
$$f_{xy} = (f_x)_y = \frac{-1}{y^2} \implies f_{xy}(1, 1) = -1$$
$$f_{yx} = (f_y)_x = \frac{-1}{y^2} \implies f_{yx}(1, 1) = -1$$
$$f_{xx} = 0 \implies f_{xx}(1, 1) = 0$$
$$f_{yy} = \frac{2x}{y^3} \implies f_{yy}(1, 1) = 2$$

2. Compute $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$ for the following functions at the indicated point.

   (a) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}; (3, 0, 4)$

Solution:

$$f_x(x,y,z) = \frac{1}{2}(x^2+y^2+z^2)^{-3/2}2x = \frac{x}{(x^2+y^2+z^2)^{3/2}}$$

Then,

$$f_x(3,0,4) = \frac{3}{(3^2+0^2+4^2)^{3/2}} = \frac{3}{25^{3/2}} = 3/125.$$

$$f_y(x,y,z) = \frac{1}{2}(x^2+y^2+z^2)^{-3/2}2y = \frac{y}{(x^2+y^2+z^2)^{3/2}}$$

Then,

$$f_y(3,0,4) = \frac{0}{(3^2+0^2+4^2)^{3/2}} = 0.$$

$$f_z(x,y,z) = \frac{1}{2}(x^2+y^2+z^2)^{-3/2}2z = \frac{z}{(x^2+y^2+z^2)^{3/2}}$$

Then,

$$f_z(3,0,4) = \frac{4}{(3^2+0^2+4^2)^{3/2}} = \frac{4}{25^{3/2}} = 4/125.$$

(b) $f(x,y,z) = \cos(xy^2) + e^{3xyz}; (\pi,1,1)$

Solution:

$$f_x(x,y,z) = -\sin(xy^2)y^2 + 3yze^{3xyz} \implies f_x(\pi,1,1) = -\sin(\pi) + 3e^{\pi} = 3e^{\pi}$$

$$f_y(x,y,z) = -\sin(xy^2)2xy + 3xze^{3xyz} \implies f_y(\pi,1,1) = -2\pi\sin(\pi) + 3\pi e^{\pi} = 3\pi e^{\pi}$$

$$f_z(x,y,z) = 0 + 3xye^{3xyz} \implies f_z(\pi,1,1) = 3\pi e^{\pi}$$

3. Compute $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, and $\frac{\partial u}{\partial z}$ for the following functions at the indicated point.

(a) $u = e^{xyz}(xy+xz)$

Solution:

$$u_x(x,y,z) = e^{xyz}(y+z) + yze^{xyz}(xy+xz) = e^{xyz}(y+z)(1+xyz)$$
$$u_y(x,y,z) = e^{xyz}x + xze^{xyz}(xy+xz) = xe^{xyz}(1+xyz+xz^2)$$
$$u_z(x,y,z) = e^{xyz}x + xye^{xyz}(xy+xz) = xe^{xyz}(1+xy^2+xyz)$$

(b) $u = e^x \cos(yz^2)$

Solution:

$$u_x(x, y, z) = e^x \cos(yz^2)$$
$$u_y(x, y, z) = -e^x \sin(yz^2) z^2$$
$$u_z(x, y, z) = -e^x \sin(yz^2) 2yz$$

4. Compute

$$\frac{\partial}{\partial \lambda} \left(\frac{\cos(\lambda\mu)}{1 + \lambda^2 + \mu^2} \right).$$

Solution:

$$\begin{aligned} \frac{\partial}{\partial \lambda} \left(\frac{\cos(\lambda\mu)}{1 + \lambda^2 + \mu^2} \right) &= \frac{(1 + \lambda^2 + \mu^2)(-\mu \sin(\lambda\mu)) - \cos(\lambda\mu)(2\lambda)}{(1 + \lambda^2 + \mu^2)^2} \\ &= \frac{-\mu \sin(\lambda\mu)}{1 + \lambda^2 + \mu^2} + \frac{-2\lambda \cos(\lambda\mu)}{(1 + \lambda^2 + \mu^2)^2} \end{aligned}$$