Score: \_\_\_\_\_

## WORKSHEET 8 - DUE 11/11

 $\begin{array}{l} {\rm MATH} \ 2110{\rm Q}-{\rm Fall} \ 2015 \\ {\rm Professor} \ {\rm Hohn} \end{array}$ 

You must show all of your work for full credit. Please circle/box your answers or write a brief sentence indicating your answer.

1. Evaluate the integral

$$\int_0^a \int_{-\sqrt{a^2 - y^2}}^0 x^2 y \, dx \, dy$$

by converting to polar coordinates.

2. We define the improper integral (over the entire plane  $\mathbb{R}^2$ )

$$\iint_{\mathbb{R}^2} e^{-(x^2 + y^2)} \, dA = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2 + y^2)} \, dA = \lim_{a \to \infty} \iint_{D_a} e^{-(x^2 + y^2)} \, dA$$

where  $D_a$  is the disk with radius a and center at the origin. Show that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dA = \pi.$$

3. Evaluate the triple integral

$$\iiint_E xy \, dV$$

where E is bounded by the parabolic cylinders  $y = x^2$  and  $x = y^2$  and the planes z = 0 and z = x + y.

4. Convert the following into an integral in cylindrical coordinates.

$$\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz \, dz \, dx \, dy$$