Name: _____

Score: _____ /15

WORKSHEET 1 - CHAPTER 12 (DUE TUES, FEB 10)

Math 2110Q – Spring 2015
Professor Hohn

You must show all of your work to receive full credit!

Answers (not necessarily in order):

$$-1/\sqrt{6}, -2, -4, 18, \langle 2, -1, 5\rangle, x = -2 + 2t, y = 2 - t, x + 1 = \frac{y+1}{3} = z - 10,$$
$$z = 4 + 5t, -4x + 3y + z = -14, \approx 417, \langle -196, 3.92\rangle, x + 3y + z = 6,$$
$$\langle 196, 3.92\rangle (1, 4, 4), x + y + z = 4, 22/\sqrt{26}, x = 2 + t, y = -t, z = 4 + 2t,$$
$$750,000\sqrt{3}, -1, 3\sqrt{35}, \langle 33, -21, 6\rangle,$$

1. Calculate the given quantity if $\vec{a} = \hat{x} + \hat{y} - 2\hat{z}$, $\vec{b} = 3\hat{x} - 2\hat{y} + \hat{z}$, $\vec{c} = \hat{y} - 5\hat{z}$.
   (a) $\vec{a} \cdot \vec{b}$

   (b) $\left\| \vec{b} \times \vec{c} \right\|$

(c) $\vec{a} \cdot (\vec{b} \times \vec{c})$

(d) $\vec{a} \times (\vec{b} \times \vec{c})$

(e) $\text{comp}_{\vec{a}}\vec{b}$

2. Find the values of x such that the vectors $\langle 3, 2, x \rangle$ and $\langle 2x, 4, x \rangle$ are orthogonal.

3. Find parametric equations for the line through $(-2, 2, 4)$ and perpendicular to the plane $2x - y + 5z = 12$.

4. Find an equation of a plane though $(3, -1, 1), (4, 0, 2)$, and $(6, 3, 1)$.

5. Find the point in which the line with parametric equations $x = 2-t, y = 1+3t, z = 4t$ intersects the plane $2x - y + z = 2$.

6. Find an equation of the plane through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and perpendicular to the plane $x + y - 2z = 1$.

7. Find the distance between the planes $3x + y - 4z = 2$ and $3x + y - 4z = 24$.

8. (a) Find an equation of the plane that passes through the points $A(2, 1, 1), B(-1, -1, 10)$, and $C(1, 3, -4)$.

(b) Find symmetric equations for the line through B that is perpendicular to the plane in part (a).

(c) A second planes passes through $(2, 0, 4)$ and has normal vector $\langle 2, -4, -3 \rangle$. Show that the acute angle between the planes is approximately 43°.

(d) Find parametric equations for the line of intersection of the two planes.

Applications

9. A tow truck drags a stalled car along a road. The chain makes an angle of 30° with the road and the tension in the chain is 1500 N. How much work is done by the truck in pulling the car 1 km? (Watch your units)

10. A wrench 30 cm long lies along the positive $y-$axis and grips a bolt at the origin. A force is applied in the direction $\langle 0, 3, -4 \rangle$ at the end of the wrench. Find the magnitude of the force needed to supply 100 N$\cdot$ m of torque to the bolt. (Watch your units)

11. A clothesline is tied between two poles, 8 m apart. The line is quite taut and has negligible sag. When a wet shirt with a mass of 0.8 kg is hung at the middle of the line, the midpoint is pulled down 8 cm. Find the tension in each half of the clothesline. (Watch your units...also, gravity)

Bonus: Challenge Question

12. Suppose $\vec{v_1}$ and $\vec{v_2}$ are vectors with $\|\vec{v_1}\| = 2$ and $\|\vec{v_2}\| = 3$, and $\vec{v_1} \cdot \vec{v_2} = 5$. Let $\vec{v_3} = \text{proj}_{\vec{v_1}} \vec{v_2}$, $\vec{v_4} = \text{proj}_{\vec{v_2}} \vec{v_3}$, $\vec{v_5} = \text{proj}_{\vec{v_3}} \vec{v_4}$, and so on. Compute $\sum_{n=1}^{\infty} \|\vec{v_n}\|$.