Name: ______________________

Score: ________ /15

WORKSHEET 2 - CHAPTER 14 (DUE TUES, FEB 24)

Math 2110Q – Spring 2015
Professor Hohn

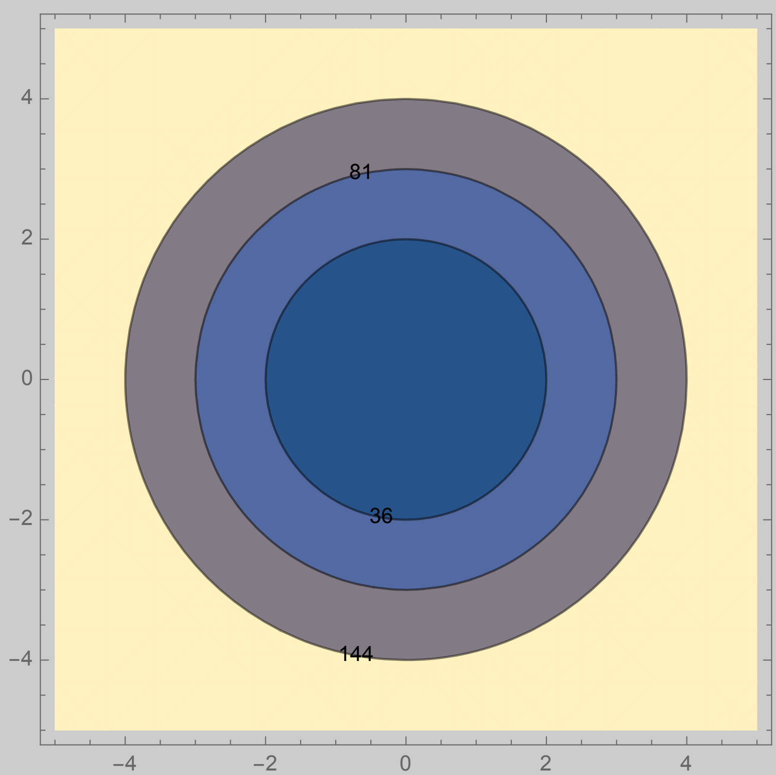
You must show all of your work to receive full credit!

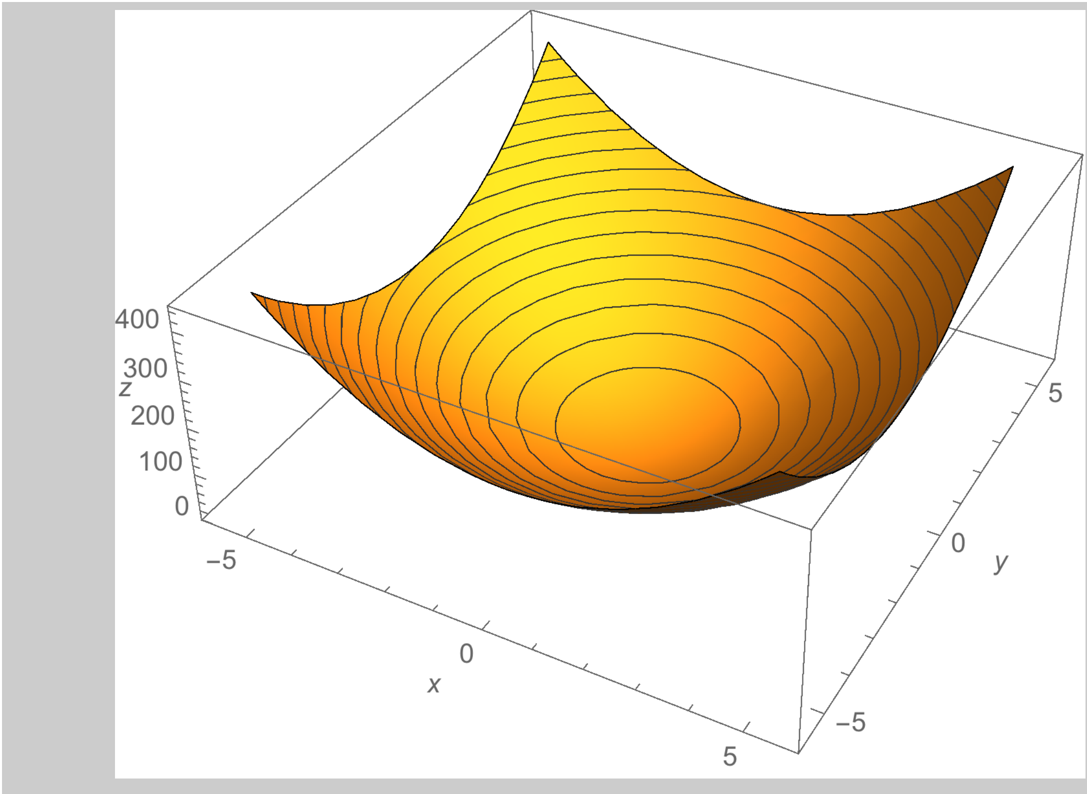
1. Sketch both a contour map (graph of level curves) and the graph of the function

$$f(x,y) = 9x^2 + 9y^2.$$

Pick at least three k values for your contour map (like $k = 36, 81, 144$).

Solution: Here is an example of a contour map with $k = 36, 81, 144$. When drawing these graphs be hand, please make the axes equally spaced.





2. The temperature at a point (x, y) on a flat metal plate is given by $T(x, y) = 60/(1 + x^2 + y^2)$, where T is measured in °C and x, y in meters. Find the rate of change of temperature with respect to distance at the point $(2, 1)$ in (a) the x-direction and (b) the y-direction.

Solution: To find the rate of change of temperature with respect to distance in the x-direction, we need to find T_x at the point $(2, 1)$.

$$T_x = \frac{-60(2x)}{(1 + x^2 + y^2)^2} = \frac{-120x}{(1 + x^2 + y^2)^2}$$

Then,

$$T_x(2, 1) = \frac{-120(2)}{(1 + 2^2 + 1^2)^2} = \frac{-240}{36} = -20/3$$

Similarly, to find the rate of change of temperature with respect to distance in the y-direction, we need to find T_y at the point $(2, 1)$.

$$T_y = \frac{-60(2y)}{(1 + x^2 + y^2)^2} = \frac{-120y}{(1 + x^2 + y^2)^2}$$

Then,

$$T_y(2, 1) = \frac{-120(1)}{(1 + 2^2 + 1^2)^2} = \frac{-120}{36} = -10/3$$

3. Let $z = \ln(e^x + e^y)$. Verify that the function is a solution of the differential equation

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1.$$

Solution: To verify that the function is a solution to the differential equation, we first need to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

$$\frac{\partial z}{\partial x} = e^x/(e^x + e^y)$$

and

$$\frac{\partial z}{\partial y} = e^y/(e^x + e^y).$$

Now, we will show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ by starting with the left-hand side of the equation and showing that it is equal to the right-hand side.

$$\begin{aligned}\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} &= e^x/(e^x + e^y) + e^y/(e^x + e^y) \\ &= (e^x + e^y)/(e^x + e^y) \\ &= 1\end{aligned}$$

Hence, the function satisfies the differential equation.

4. A visitor from planet Zorg tells you that there is a function f whose partial derivatives are $f_x(x, y) = x + 4y$ and $f_y(x, y) = 3x - y$. Should you believe the visitor? Why or why not?

Solution: There are a couple ways to approach this question.

Method 1: From our talk about Clairaut's Theorem, we know that $f_{xy} = f_{yx}$ for continuous functions f_{xy} and f_{yx}. Therefore, if this isn't true for the equations given by the visitor, we can say that we don't believe our alien. $f_{xy} = 4$ and $fyx = 3$. Thus, $f_{xy} \neq f_{yx}$, and we can say that we don't believe our visitor.

Method 2: We can try to find our original function f by integrating f_x with respect to x and then taking a partial derivative with respect to y. We can check to see if our f_y matches our visitor's function f_y.

$$\int f_x dx = \int (x + 4y)dx = \frac{x^2}{2} + 4xy + K(y) + C \implies f(x, y) = \frac{x^2}{2} + 4xy + K(y) + C$$

where K is a function of y. Note that this is because when we took a partial derivative any function of y acts like a constant. Now, we can look at our function f we created and take a derivative with respect to y.

$$f_y = 0 + 4x + K'(y) \implies f_y = 4x + K'(y) \neq 3x - y.$$

Note that $4x \neq 3x$. Hence, no function f exists with the partial derivatives mentioned by the visitor.

5. Find u_{xy} and u_{yx} where $u = \ln(x + 2y)$.

Solution: Let's start by finding u_x and u_y.

$$u_x = \frac{1}{x+2y} = (x+2y)^{-1}$$

$$u_y = \frac{1}{x+2y} \cdot 2 = \frac{2}{x+2y} = 2(x+2y)^{-1}$$

Now, we will take do $(u_x)_y$.

$$u_{xy} = -1(x+2y)^{-2} \cdot 2 = -2(x+2y)^{-2} = \frac{-2}{(x+2y)^2}$$

Then,

$$u_{yx} = -2(x+2y)^{-2} = \frac{-2}{(x+2y)^2}$$

Notice that $u_{xy} = u_{yx}$.