Name: ______________________

Score: ________ /15

WORKSHEET 4 - CHAPTER 14 (DUE TUES, MAR 10)

Math 2110Q – Spring 2015
Professor Hohn

You must show all of your work to receive full credit!

Solutions (in no particular order):

$$\frac{1}{3}x + y, 5, 0, \langle 1+2t, 1+2t, 1+2t\rangle, \frac{e^{2t}}{1+se^{2t}}ds + \frac{2se^{2t}}{1+se^{2t}}dt, 4xe^{-2y}, 0,$$
$$-\sqrt{2}, -2e^{-2y}, \frac{2x^3}{x^2+y^2} + 2x\ln(x^2+y^2), (3,0), (0,0), \{(x,y) \mid y > -x-1\}, \sqrt{6},$$
$$\langle -1/\sqrt{6}, -1/\sqrt{6}, -2/\sqrt{6}\rangle, \frac{2x^2y}{x^2+y^2}, (1,1), (1,2), (0,3), (2,4), x+y+z=3, 25/6, \sqrt{2}$$

1. Find and sketch the domain of the function $f(x,y) = \ln(x+y+1)$.

2. Sketch 3 level curves of the function $f(x, y) = \sqrt{4x^2 + y^2}$. Try $k = 0, 2, 4$.

3. Find the first partial derivatives of $f(x, y) = x^2 \ln(x^2 + y^2)$.

4. Find all second partial derivatives of $f(x, y) = xe^{-2y}$.

5. Verify the linear approximation $\sqrt{y + \cos^2 x} \approx 1 + \frac{1}{2}y$ at the point $(0, 0)$.

6. Find the linearization $L(x, y)$ of the function $f(x, y) = y + \sin(x/y)$ at the point $(0, 3)$.

7. Suppose $xy + yz + zx = 3$.

(a) Find the tangent plane at the point $(1, 1, 1)$.

(b) Find the normal line to the given surface at the point $(1, 1, 1)$.

8. Find du if $u = \ln(1 + se^{2t})$.

9. Suppose $v = x^2 \sin y + ye^{xy}$, where $x = s + 2t$ and $y = st$.

(a) Use the Chain Rule to find $\partial v/\partial s$ when $s = 0$ and $t = 1$.

(b) Use the Chain Rule to find $\partial v/\partial t$ when $s = 0$ and $t = 1$.

10. Find the directional derivative of $f(x, y, z) = x^2y + x\sqrt{1+z}$ at the point $(1, 2, 3)$ in the direction $\langle 2, 1, -2 \rangle$.

11. Find the maximum rate of change of $f(x, y, z) = (x + y)/z$ at the point $(1, 1, -1)$ and the direction in which it occurs.

12. Find the local maximum and minimum values and saddle points of the function

$$f(x, y) = 3xy - x^2y - xy^2.$$

13. Find the absolute maximum and minimum values of $f(x, y) = 4xy^2 - x^2y^2 - xy^3$ on the closed triangular region in the xy-plane with vertices $(0, 0)$, $(0, 6)$, $(6, 0)$.

14. Use Lagrange Multipliers to find the maximum and minimum values of $f(x,y) = \frac{1}{x} + \frac{1}{y}$ given the constraint $\frac{1}{x^2} + \frac{1}{y^2} = 1$.

Bonus

15. A rectangle with length L and width W is cut into four smaller rectangles by two lines parallel to the sides. Find the maximum and minimum values of the sum of the squares of the area of the smaller rectangles.