

Name: \_\_\_\_\_

Score: \_\_\_\_\_ /15

## WORKSHEET 6 - CHAPTER 15 (DUE TUES, APR 7)

Math 2110Q – Spring 2015  
Professor Hohn

You must show all of your work to receive full credit!

1. Calculate the value of the integral

$$\iint_D x \, dA$$

where  $D$  is the region in the first quadrant between the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 2$ .

2. Evaluate the integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} \, dy \, dx$$

by converting to polar coordinates.

3. Evaluate the integral

$$\int_0^a \int_{-\sqrt{a^2-y^2}}^0 x^2 y \, dx \, dy$$

by converting to polar coordinates.

4. We define the improper integral (over the entire plane  $\mathbb{R}^2$ )

$$\iint_{\mathbb{R}^2} e^{-(x^2+y^2)} dA = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dA = \lim_{a \rightarrow \infty} \iint_{D_a} e^{-(x^2+y^2)} dA$$

where  $D_a$  is the disk with radius  $a$  and center at the origin. Show that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dA = \pi.$$

5. Use a double integral to find the area of the region inside the cardioid  $r = 1 + \cos \theta$  and outside the circle  $r = 3 \cos \theta$ .