Linear Algebra Exercises

Math 2410Q – Fall 2013

Professor Hohn

1. Let A and B be the 3×3 matrices

$$A = \begin{pmatrix} 4 & 1 & -1 \\ -2 & 0 & 3 \\ -3 & 2 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} -4 & 1 & 1 \\ 0 & -1 & 2 \\ -3 & 0 & 4 \end{pmatrix}.$$

   (a) Find $A + B$.

   (b) Find $2A - 3B$.

   (c) Find AB.

   (d) Find BA.

2. Let M be the 2×2 matrix

$$M = \begin{pmatrix} 5 & 1 \\ -2 & 2 \end{pmatrix}.$$

(a) Find the eigenvalues of M.

(b) Find the corresponding eigenvectors of M.

3. Let K be the 2×2 matrix

$$K = \begin{pmatrix} -5 & 1 \\ 0 & -4 \end{pmatrix}.$$

(a) Find the eigenvalues of K.

(b) Find the corresponding eigenvectors of K.

4. Let A and A^{-1} (the inverse matrix of A) be the 2×2 matrices

$$A = \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}, \qquad A^{-1} = \begin{pmatrix} -\frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{pmatrix}.$$

(a) Find AA^{-1}.

(b) Compute the eigenvalues and eigenvectors of A.

(c) Compute the eigenvalues and eigenvectors of A^{-1}.

(d) Compare the eigenvalues and eigenvectors of A to the eigenvalues and eigenvectors of A^{-1}.

5. Let B and B^2 be the 2×2 matrices

$$B = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}, \quad B^2 = \begin{pmatrix} 7 & -3 \\ -2 & 6 \end{pmatrix}.$$

(a) Compute the eigenvalues and eigenvectors of B.

(b) Compute the eigenvalues and eigenvectors of B^2.

(c) Compare the eigenvalues and eigenvectors of B to the eigenvalues and eigenvectors of B^2.