Midterm 1 - Preview

Math 2710 – Spring 2014
Professor Hohn

Instructions: These exercises are to be worked on alone! You may use your notes and the textbook, but you are **not allowed to ask for, receive, nor give others assistance on these exercises.**

1. Let a and b be integers. Show that if $\gcd(a, b) = 1$, then $\gcd(a, a + b) = 1$.
2. Prove or find a counterexample. Let $a, b, c \in \mathbb{Z}$.
   (a) If $a \mid b$ or $a \mid c$, then $a \mid bc$.
   (b) If $a \mid bc$, then $b \mid a$ or $c \mid a$.
3. Prove that if a and b are nonzero integers, then
$$\operatorname{lcm}(a, b) = \frac{|ab|}{\gcd(a, b)}.$$
4. Prove or give a counterexample. If x and y are real numbers, then $\forall x, \exists y\, (x^2 > y^2)$.
5. Show that the real number $\sqrt{2}$ is irrational. (Hint: Suppose $\sqrt{2}$ is rational, and show a contradiction.)
6. Let m be an integer greater than 1 and $a, b \in \mathbb{Z}$. Suppose $m \mid ab \implies m \mid a$ or $m \mid b$. Show m is prime.
7. Prove that $a^2 \mid b^2$ if and only if $a \mid b$.
8. Prove or give a counterexample. Let S and T be sets. $S \cup T = T \iff S \subseteq T$.
9. Show that the Diophantine equation $ax^2 + by^2 = c$ does not have any integer solutions unless $\gcd(a, b) \mid c$. If $\gcd(a, b) \mid c$, does the equation always have an integer solution?
10. Prove or give a counterexample. If x is a real number such that $x^4 + 2x^2 - 2x < 0$, then $0 < x < 1$.