Homework 12 (Due Tues, Apr 29)

Math 2710 – Spring 2014
Professor Hohn

Using the proof techniques we have learned in class, prove each statement.

1. **Statements to think about:**

   True or False. Justify your answer.

   (a) If $\{s_n\}$ is a sequence and $s_i = s_j$, then $i = j$.

   (b) If $s_n \to s$, then for every $\varepsilon > 0$ there exists $N \in \mathbb{R}$ such that $n > N$ implies $|s_n - s| < \varepsilon$.

   (c) If for every $\varepsilon > 0$ there exists $N \in \mathbb{R}$ such that $n > N$ implies $s_n < \varepsilon$, then $s_n \to 0$.

   (d) If $s_n \to k$ and $t_n \to k$ then $s_n = t_n$ for all $n \in \mathbb{N}$.

2. * Show that $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$. (Hint: Given any $\varepsilon > 0$, we have to find N such that $n > N$ implies that $\frac{1}{\sqrt{n}} < \varepsilon$.)

3. * Prove that if a sequence converges, its limit is unique. (Hint: Suppose that the sequence $\{s_n\}$ has two different limits s and t, and show that $s = t$ by showing that $|s - t| < \varepsilon$. You will need the triangle inequality: $|x + y| \leqslant |x| + |y|$.)

4. * Suppose that $\lim s_n = 0$. If $\{t_n\}$ is a bounded sequence, prove that $\lim(s_n t_n) = 0$.