Homework 1 (Due Tues, Jan 28)

Math 2710 – Spring 2014

Professor Hohn

Using the proof techniques we have learned in class, prove or give a counterexample to each statement.

1. Let a and b be real numbers. Prove that if $ab = 0$, then $a = 0$ or $b = 0$.

Solution:

Proof. (Proof by contradiction) Suppose $ab = 0$, and $a \neq 0$ and $b \neq 0$. Since $a \neq 0$, we can divide both sides of the equation $ab = 0$ by a. As a result, we have $b = 0$ ↯ (Contradiction). Therefore, if $ab = 0$, then $a = 0$ or $b = 0$. □

2. $\forall x \in \mathbb{R}$, $(x^2 + 5x + 7 > 0)$. (Note that $\forall$ means "for all." The statement reads, "For all x in the real numbers, $(x^2 + 5x + 7 > 0)$.")

Solution:

Proof. For all $x \in \mathbb{R}$,

$$\begin{aligned} x^2 + 5x + 7 &= \left(x^2 + 5x + \frac{25}{4}\right) - \frac{25}{4} + 7 \\ &= \left(x + \frac{5}{2}\right)^2 + \frac{3}{4}. \end{aligned}$$

Since for all $x \in \mathbb{R}$, $\left(x + \frac{5}{2}\right)^2 \geq 0$, and $\frac{3}{4} > 0$, $\left(x + \frac{5}{2}\right)^2 + \frac{3}{4} > 0$. Therefore, $x^2 + 5x + 7 > 0$. □

3. If m and n are integers with mn odd, then m and n are odd.

Solution:

Proof. (Proof by contrapositive) We want to show that if m or n is even, then mn is even. Without loss of generality, let m be even. Then, $m = 2k$ for some integer k, and

$$\begin{aligned} mn &= (2k)n \\ &= 2(kn) \end{aligned}$$

where kn is an integer. Hence, mn is divisible by 2 and is even.

□