Homework 4 (Due Tues, Feb 18)

Math 2710 – Spring 2014
Professor Hohn

Using the proof techniques we have learned in class, prove each statement.

1. **Multiplying an integer by zero.**

   Let $a, b, c \in \mathbb{Z}$. *Then,* $0 \cdot a = 0$.

   Solution:

   Proof. Let $a \in \mathbb{Z}$. Then,

   $$\begin{aligned} 0 \cdot a &= (0 + 0) \cdot a \\ &= 0 \cdot a + 0 \cdot a \quad \text{by the distributive property} \end{aligned}$$

   Subtracting $0 \cdot a$ from both sides of the equation, we have $0 = 0 \cdot a$. □

2. **Multiplying by a negative integer**.

   Let $a, b, c \in \mathbb{Z}$. *Then,* $a \cdot (-b) = -ab$.

   Solution:

   Proof. Let $a, b \in \mathbb{Z}$. Then,

   $$\begin{aligned} a \cdot 0 &= a \cdot (b + -b) \quad b \text{ and } -b \text{ are additive inverses} \\ &= ab + a(-b) \quad \text{by distributive property} \end{aligned}$$

   Thus, by the previous proof, $0 = ab + a(-b)$ and $-ab = a(-b)$. □

3. **Cancellation law**

Let $a, b, c \in \mathbb{Z}$. Then, if $a \neq 0$ and $ab = ac$, then $b = c$.

Solution:

Proof. Let $a, b, c \in \mathbb{Z}$, $a \neq 0$, and $ab = ac$. Then,

$$\begin{aligned} ab - ac &= 0 \\ a(b - c) &= 0 \end{aligned}$$

Since $a \neq 0$, $b - c = 0$ and $b = c$.

□