Homework 6 (Due Tues, March 11)

Math 2710 – Spring 2014

Professor Hohn

Using the proof techniques we have learned in class, prove each statement.

1. Let A, B, C be sets. Prove the following:
   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (b) $A - (B \cup C) = (A - B) \cap (A - C)$
   (c) $(A \cup B)' = A' \cap B'$

2. Let A be a set and $\{B_i\}_{i \in I}$ an indexed family of sets. Prove the following:
$$\left(\bigcup_{i \in I} B_i \right) \cap A = \bigcup_{i \in I} (B_i \cap A).$$

3. * Let A be a set and let $\{B_i\}_{i \in I}$ be an indexed family of sets. Prove the following is true.
$$A - \bigcup_{i \in I} B_i = \bigcap_{i \in I} (A - B_i)$$

4. Show that if $A \subset B$ then $\mathcal{P}(A) \subset \mathcal{P}(B)$. ($\mathcal{P}(A)$ is the power set of A.)

5. Let $S = \{1, 2, 3\}$. In each case, give an example of a relation R on S that has the stated properties.
   (a) R is not symmetric, not reflexive, and not transitive.
   (b) R is transitive and reflexive, but not symmetric.

6. * A relation R is *antisymmetric* if xRy and yRx together imply that $x = y$. A relation R on S is a partial ordering if R is reflexive, antisymmetric, and transitive. For example, the relation "$\leqslant$" on $\mathbb{R}$ is a partial ordering . Show that each of the following is a partial ordering.
   (a) The inclusion relation "$\subseteq$" on the power set of a given set A.
   (b) The divisibility relation on $\mathbb{N}$. (If $a, b \in \mathbb{N}$, define $a \mid b$ to mean that $b = a \cdot q$ for some $q \in \mathbb{N}$.)

7. Determine each of the following sets.
   (a) $\mathcal{P}(\{2\})$
   (b) $\mathcal{P}(\mathcal{P}(\{2\}))$
   (c) $\mathcal{P}(\mathcal{P}(\mathcal{P}(\{2\})))$

8. Given any two sets S and T, the *Cartesian product* of S and T is the new set $S \times T$ defined by

$$S \times T = \{(s,t) : s \in S, t \in T\}.$$

If S and T are sets, $A \subseteq S$ and $B \subseteq T$, prove that $A \times B \subseteq S \times T$.

9. * Prove that A and B are disjoint if and only if $A \subseteq B'$.

10. Prove or give a counterexample to the assertion $A \cup (B - A) = B$.