Score: ________

Math 2710 – Spring 2014 Name: ____________________

Quiz 4 – 10 min

Answer each of the following questions. You must **show your work** to receive full credit!

1. Prove that $\gcd(a, c) = \gcd(b, c) = 1$ if and only if $\gcd(ab, c) = 1$.

Solution: ($\Longrightarrow$) Suppose the $\gcd(a, c) = \gcd(b, c) = 1$. Then, $\exists p, q \in \mathbb{Z}$ such that $bp + cq = 1$. Multiplying both sides by a, we have $ab \cdot p + c \cdot aq = a$. By the Linear Diophantine Theorem, $\gcd(ab, c) \mid a$. By the Euclidean Algorithm, $\exists x, y \in \mathbb{Z}$ such that $ax + cy = 1$. Since $\gcd(ab, c) \mid a$ and $\gcd(ab, c) \mid c$, $\gcd(ab, c) \mid (ax + cy)$ and $\gcd(ab, c) \mid 1$. Hence, $\gcd(ab, c) = 1$.

($\Longleftarrow$) On the other hand, if $\gcd(ab, c) = 1$, then $\exists x, y \in \mathbb{Z}$ such that $ab \cdot x + c \cdot y = 1$. Then, $a(bx) + c(y) = 1$ and $\gcd(a, c) = 1$. In addition, $b(ax) + c(y) = 1$ and $\gcd(b, c) = 1$. Hence, $\gcd(a, c) = \gcd(b, c) = 1$.