Extra Homework Exercises

Math 3160 – Fall 2015
Professor Hohn

List of supplementary exercises for Chapters 4 and 5.

1. Suppose that the number of earthquakes per year in a certain region in the US is well-modeled by a Poisson random variable with an average of 3 earthquakes occurring per year. Calculate the probability that in a given year there are at least 4 earthquakes in this region, given that there are at least 2 earthquakes.

2. Let X be a random variable with CDF given by

$$F_X(t) = \begin{cases} 0 & t < -1 \\ \frac{1}{2} & -1 \leq t < 1 \\ \frac{1}{2}t & 1 \leq t < 2 \\ 1 & t \geq 2 \end{cases}$$

   Calculate $\mathbb{E}[X]$.

3. Let $X \stackrel{d}{=} \text{Bin}(4, 1/3)$. What is $P(X^2 + X \geq 1)$?

4. Let $X \stackrel{d}{=} \text{Bin}(4, 1/3)$ and $Y \stackrel{d}{=} \text{Geom}(1/2)$. For each choice of Z, find the state space S_Z of Z and calculate $\mathbb{E}[Z]$:

   1. $Z = Y - X$.
   2. $Z = X^2 + 3Y$.

5. Use the differentiation trick

$$\int_0^\infty t^n e^{-\lambda t}\, dt = (-1)^n \left(\frac{\partial}{\partial \lambda}\right)^n \int_0^\infty e^{-\lambda t}\, dt$$

   to find a general formula (depending on n) for $\int_0^\infty t^n\, e^{-\lambda t}\, dt$. Note that $\left(\frac{\partial}{\partial \lambda}\right)^n$ is just notation meaning the nth derivative with respect to λ; another way this is commonly written is $\frac{\partial^n}{\partial \lambda^n}$. Use this to find $\mathbb{E}[X^5]$ where $X \stackrel{d}{=} \text{Exp}(2)$.