PRACTICE PROBLEMS FOR EXAM 1

Math 3160Q – Fall 2015
Professor Hohn

Below is a list of practice questions for Exam 1. Any quiz, homework, or example problem has a chance of being on the exam. For more practice, I suggest you work through the review questions at the end of each chapter as well.

1. As Mario is walking, he stumbles upon three, large colored sewer pipes: Pipe I, Pipe II, and Pipe III. After some inspection, he finds that inside Pipe I, there are 10 red coins and 7 yellow coins. In Pipe II, he finds 8 red and 8 yellow coins. In Pipe III, he finds 2 red and 10 yellow coins. Luigi, who enjoys collecting coins, sees Mario and jaunts over to the three pipes. Mario (who knows Luigi well) believes that Luigi would jump into Pipe I with probability 0.3, the Pipe II with probability 0.2, and Pipe III with probability 0.5.
   (a) While Mario is checking out the surrounding area for possible threats (Koopas...ahhh!!), Luigi jumps into one of the pipes and returns with a coin. If Luigi is equally likely to grab any coin, what is the probability that Luigi picked a red coin?
   (b) Given Luigi picked a red coin, what is the probability that the coin came from Pipe II?

2. An urn initially contains 5 red marbles and 7 blue marbles. Bored out of your mind, you decide to play a "game" that goes as follows. During each round, you randomly pull a marble out of the urn. If the marble you chose was red, you return the marble back into the urn along with 1 more blue marble. If the marble you chose was blue, you put the marble back into the urn along with 2 more red marbles. What is the probability that on the first round you drew a red marble and on the third round you drew a blue marble?

3. Your professor hands you three coins. The first coin has a probability of .6 of landing heads; the second coin has a probability .3 of landing heads; the third coin has a probability .5 of landing heads (i.e., the third coin is fair). You are asked to conduct the following experiment: You flip the first coin and note the result (heads or tails). If the first flip resulted in heads, you then flip the second coin and note the result. If, on the other hand, the first flip resulted in a tails, you then flip the third coin and note the result.
   (a) What is a reasonable sample space Ω for this experiment?
   (b) Let X be (the random variable giving) the number of heads which occurred during the experiment. What is the state space S_X of X?
   (c) If X is the same random variable as in the previous part and F_X is the cumulative distribution function (CDF) of X, what is $F_X(1.5)$?

4. A circuit has three nodes: A, B, and C. Each node is *independently* functional with probability p_A, p_B, and p_C respectively. The circuit works if either A is functional, or both B and C are functional. Otherwise, the circuit does not work. Find the probability that the circuit is working. (Your answer should be in terms of p_A, p_B, and p_C. This problem is not meant to be tricky!)

5. After watching Guardians of the Galaxy, our class became extremely motivated to befriend a raccoon. In this class there are 20 males and 16 females.
   (a) We are going to make two committees from the people in this class. The first committee will consist of 6 people, 3 males and 3 females, to search for a raccoon to befriend; the second committee will consist of 4 other people, 2 males and 2 females, to wait near a phone and medical supplies so that after the first committee actually finds a raccoon and realizes that they're vicious beasts, they can be bandaged up and an ambulance can be called. How many ways are there for us to create these committees?
   (b) After everyone survived the "raccoon incident" (and I lost my job), all 36 people in this class return to the classroom and shake everyone else's hands (no repeated handshakes) on a job well done. How many handshakes took place?

6. Mr. Tom Riddle finally bought a computer, and he's looking to stay connected with his Death Eater friends on Instagram. He wants to create his new screen name by rearranging the letters

   MRTOMRIDDLE

   (a) How many different screen names (distinguishable letter arrangements) can he make such that M is the first and fifth letter in the screen name?
   (b) Let M be the event that M is the first and fifth letter in the screen name, and let D be the event that the D's must be next to each other. What is the probability of $P(M \cap D)$?

7. You conduct an "experiment" where you continually flip a coin (keeping track of the outcome of each flip) until the either the first heads appears or the coin has been flipped 5 times, at which point you stop.
   (a) What is a reasonable *sample space* for this experiment?
   (b) If E is the event that you flip the coin an odd number of times before stopping, describe the event E as a subset of the sample space. (For example: If a sample space is $\{1, 2, 3\}$ and F is the event that the outcome is even, then the description of F as a subset of $\{1, 2, 3\}$ is just $F = \{2\}$).

8. There are three urns: Urn A, Urn B, and Urn C. Urn A contains 4 red marbles and 6 blue marbles; Urn B contains 3 red marbles and 7 blue marbles; Urn C has 8 red marbles and 2 blue marbles. You are going to select a random marble from one of the urns; there is a 1/2 probability you will draw the marble from Urn A; there is a 3/10 probabilty you will draw a marble from Urn B; there is a 1/5 probability you will draw the marble from Urn C.
   (a) Let R be the event that you select a red marble. Find $P(R)$.
   (b) Given that the marble you draw is red, what is the probability that you drew the marble from Urn A?

9. (a) How many distinguishable letter arrangements can be made from the word

   BEYONCE

   so that the E s are *not* next to each other?
   (b) (Jay-Z is lending g's...) Jay-Z has \$10 which he is going to distribute amongst his 3 friends. How many ways can he distribute the money among them if each is to receive at least \$1? We assume here that he will distribute in dollar increments.

10. An alien classroom has 29 students: 17 glargons and 12 bubuus. A committee of 2 students is to be selected.
    (a) For $k = 0, 1$, or 2, what is the number of ways to choose this committee if k of the students are to be bubuus? Note: You can either write out for each case $k = 0$, $k = 1$, and $k = 2$ explicitly, or you can instead just write one general formula in terms of k.
    (b) Suppose that each possible committee is equally likely. Let X be (the random variable giving) the number of bubuus selected for the committee. Find the probability mass function p_X of X.

11. You are lost wandering through a desert with no water and you come upon the a lovely sorceress named Beyonce. Beyonce has 3 bottles in a box to her left: Bottle #1 containing 1 liter of water, bottle #2 containing 2 liters of water, and bottle #3 containing 3 liters of water. Beyonce will ask you a first question which you have a 50% chance of getting correct. If you get the answer wrong then she disappears along with all her water bottles and you get nothing. If you get the answer correct, then you are given bottle #1 and she will ask you a second question which you have a 30% chance of getting correct. If you get the second question wrong then she disappears along with her remaining two bottles (you are left with only bottle #1 which you won in the first round). If you get the second question correct, then she gives you bottle #2 (so you now have bottles #1 and #2) and asks you a third question which you have a 10% chance of getting correct. If you get the third question wrong, then she disappears and takes the third bottle of water with her (you are left with bottles #1 and #2 which you won during the first two rounds). If you get the third question correct, then she gives you bottle #3 (so you now have bottles #1, #2, and #3) and she vanishes.
    (a) Let X be (the random variable giving) the *liters of water* that you receive in your encounter with Beyonce. What is a good and reasonable state space S_X for the random variable X?
    (b) Let E be the event that you received bottles #1 and #2, but not bottle #3. Find $P(E)$.
    (c) Let k be the value in the state space S_X such that $\{X = k\}$ is the same event as E (where X and E are the same as the in the previous two parts). What is k?

12. Hogwarts School was invited to play at an international quidditch competition. At the school, Hogwarts has 12 Chasers, 8 Beaters, 4 Keepers, and 4 Seekers of which to choose 7 players (3 Chasers, 2 Beaters, 1 Keeper, and 1 Seeker) for the all-star team. Note that the team of 7 players will be picked in such a way that no chosen player changes position (e.g. a Chaser can only be a Chaser on the team). Assume that Lily is one of the four Seekers and her brother Albus is one of the twelve Chasers.
    (a) How many different all-star teams can be formed where Lily is picked as Seeker?
    (b) Suppose Lily is fighting with her brother Albus, and they refuse to play on the same team together. How many all-star teams can be formed such that Lily and Albus do not both end up being chosen? (Note that it is possible that Lily is not picked as the Seeker in this scenario).

13. Little Red is trying to get to her grandmother's house, but the Wolf is trying to stop her! To get to her grandmother's house, Little Red walks down a road until the road splits. The split forces her to either go left or right. If she goes to the left she must *independently* find either a cloak of invisibility or a sword (or both) to get past the Wolf and get to her grandmother's house. If she goes right she must find the Wolf-Off! spray to get past the Wolf and get to her grandmother's house.

Assume that Little Red has a probability p_L of going left at the split in the road, and p_R of going right. Given that Little Red goes left at the split, she has a probability p_C to find the cloak of invisibility and probability p_S to find the sword. Given that she goes right at the split, then she has a probability of p_W of finding the Wolf-Off! spray.

Question: What is the probability that Little Red makes it to her grandmother's house?

14. The following parts refer to the letters: LAMEFIREALARM. Recall that "word" means *distinguishable* letter arrangements.
    (a) How many words can be made with the above letters such that the M's are *not* next to each other?
    (b) What is the probability that, given a random word made from the letters above, the M's will be separated by at least two letters?

15. Suppose you shuffle together two standard 52 card decks (effectively, you double the number of each card). You are randomly dealt a 5 card poker hand. Let X be the random variable counting the number of 2's in your hand.
    (a) What is the state space S_X of X?
    (b) What is the probability mass function p_X of X? That is, find $p_X(k) = P(X = k)$ for k in S_X.
    (c) What is the value of the distribution F_X of X at 5.1? That is, find $F_X(5.1) = P(X \leq 5.1)$.

16. Marble 1, marble 2, and marble 3 are in an urn. You select one of the three marbles out of the urn in such a way that you have a 40% chance of selecting marble 1, a 50% chance of selecting marble 2, and a measly 10% chance of selecting marble 3. After you've selected the marble you then roll fair dice in the following way: if you selected marble 1, you roll one fair die; if you selected marble 2, you roll two fair dice; if you selected marble 3, you roll three fair dice. Given that the sum of the outcomes of the dice you roll is 3, what is the probability that you selected marble 2? (To make sure we're clear: If you roll three dice and the outcomes are i, j, and k, then the sum of the outcomes is $i + j + k$. If you roll two dice and the outcomes are i and j, then the sum of the outcomes is $i + j$. If you roll one die and the outcome is i, then the sum of the outcomes is i.)

17. You play a game where during each round you win either \$1 with probability $2/3$ or you win \$3 with probability $1/3$. Assume that the outcome of each round is independent of the outcomes of any other rounds. Your strategy is that you will play until the first time your winnings total at least \$4 and then you will quit. For example, if on the first two rounds you won \$1 and the third round you won \$3, then you will quit since your total winnings are \$5. Let X be the random variable representing your total winnings when you quit playing.
    (a) What is the state space of X?
    (b) Find $P(X = 4)$.
    (c) If F_X is the cumulative distribution function of X, what is $F_X(4.5)$?
    (d) If N is the random variable representing the number of rounds you played before quitting, briefly explain why $P(X = 5 \mid N = 2) = 0$.

18. Your jar of Bertie Bott's Every Flavour Beans contains only 2 flavors: 8 flavored like Strawberry and Peanut-Butter Ice Cream (SPB) and 4 flavored like Lobster (L). You play the following game with your friend James (who loves SPB): If James draw a SPB, he eats it and you add 3 L to the jar. If James draws an L, you eat it and you add 1 SPB to the jar.

(a) What is the probability that James draws 3 SPB in a row?

(b) What is the probability that on James' second draw, he draws L?