Extra Homework Exercises

Math 3160 – Spring 2015
Professor Hohn

List of supplementary exercises for Chapters 4 and 5.

Week 8

1. Let X be a random variable with CDF given by

$$F_X(t) = \begin{cases} 0 & t < -1 \\ \frac{1}{2} & -1 \le t < 1 \\ \frac{1}{2}t & 1 \le t < 2 \\ 1 & t \ge 2 \end{cases}$$

   Calculate $\mathbb{E}[X]$.

2. Suppose that the number of earthquakes per year in a certain region in the US is well-modeled by a Poisson random variable with an average of 3 earthquakes occurring per year. Calculate the probability that in a given year there are at least 4 earthquakes in this region, given that there are at least 2 earthquakes.

Week 9

Spring break!

Week 10

3. Let $X \overset{d}{=} \text{Bin}(4, 1/3)$. What is $P(X^2 + X \ge 1)$?

4. Let $X \overset{d}{=} \text{Bin}(4, 1/3)$ and $Y \overset{d}{=} \text{Geom}(1/2)$. For each choice of Z, find the state space S_Z of Z and calculate $\mathbb{E}[Z]$:
   (a) $Z = Y - X$.
   (b) $Z = X^2 + 3Y$.

Week 11

5. Use the differentiation trick

$$\int_0^\infty t^n e^{-\lambda t}\, dt = (-1)^n \left(\frac{\partial}{\partial \lambda}\right)^n \int_0^\infty e^{-\lambda t}\, dt$$

   to find a general formula (depending on n) for $\int_0^\infty t^n\, e^{-\lambda t}\, dt$. Note that $\left(\frac{\partial}{\partial\lambda}\right)^n$ is just notation meaning the nth derivative with respect to λ; another way this is commonly written is $\frac{\partial^n}{\partial \lambda^n}$. Use this to find $\mathbb{E}[X^5]$ where $X \overset{d}{=} \text{Exp}(2)$.

6. Recall that we showed in class
$$\int_{-\infty}^{\infty} e^{-t^2/2}\, dt = \sqrt{2\pi}\,.$$
Use the same idea to show that
$$\int_{-\infty}^{\infty} e^{-\alpha\, t^2}\, dt = \sqrt{\frac{\pi}{\alpha}}\,.$$

7. Use the differentiation trick
$$\int_{-\infty}^{\infty} t^{2n} e^{-\alpha t^2}\, dt = (-1)^n \left(\frac{\partial}{\partial \alpha}\right)^n \int_{-\infty}^{\infty} e^{-\alpha t^2}\, dt$$
to find a general formula (depending on n) for $\int_{-\infty}^{\infty} t^{2n}\, e^{-\alpha t^2}\, dt$. Note that $\left(\frac{\partial}{\partial \alpha}\right)^n$ is just notation meaning the nth derivative with respect to α; another way this is commonly written is $\frac{\partial^n}{\partial \alpha^n}$. Use this to find $\mathbb{E}[Z^4]$ where Z is a standard normal random variable.

Notational Hint: Notice that if you multiply together all the odd numbers $1 \cdot 3 \cdot 5 \cdots (2N-1)$, you can rewrite this as $\prod_{k=0}^{N-1} (1+2k)$. However, there is also a commonly accepted "double factorial" notation $(2N-1)!!$ which means exactly the product of all the positive odd numbers up to $2N-1$ (the double factorial also works for even numbers if the number you write the double factorial against is even). So,
$$1 \cdot 3 \cdot 5 \cdots (2N-1) = \prod_{k=0}^{N-1} (1+2k) = (2N-1)!!.$$

Week 14

8. Suppose that X and Y are jointly distributed and discrete with joint mass function
$$p_{X,Y}(s,t) = \begin{cases} c\,(s+t) & s \in \{2,3,4,5\}, t \in \{1, ..., s\} \\ 0 & \text{otherwise} \end{cases}$$
Find c and justify whether or not X and Y are independent.