PRACTICE PROBLEMS FOR EXAM 1

Math 3160Q – Spring 2015
Professor Hohn

Below is a list of practice questions for Exam 1. Any quiz, homework, or example problem has a chance of being on the exam. For more practice, I suggest you work through the review questions at the end of each chapter as well.

1. Exams at the Starfleet Academy are scored as either Pass or Fail. Jean Luc Picard must take exactly one of three exams : Quantum Field Theory (QFT), General Relativity (GR), or Underwater Basket Weaving (UBW). He has a 60% chance of taking the QFT exam, which he has a 50% chance of passing; he has a 30% chance of taking the GR exam, which he has a 70% chance of passing; he has a measly 10% chance of taking the UBW exam, which he has a 90% chance of passing.
   (a) What is the probability that Jean Luc passes his exam?
   (b) Given that he passes his exam, what is the probability that he took the QFT exam?

2. An urn initially contains 5 red marbles and 7 blue marbles. Bored out of your mind, you decide to play a "game" that goes as follows. During each round, you randomly pull a marble out of the urn. If the marble you chose was red, you return the marble back into the urn along with 1 more blue marble. If the marble you chose was blue, you put the marble back into the urn along with 2 more red marbles. What is the probability that on the first round you drew a red marble, and on the third round you drew a blue marble?

3. Your professor hands you three coins. The first coin has a probability of .6 of landing heads; the second coin has a probability .3 of landing heads; the third coin has a probability .5 of landing heads (i.e., the third coin is fair). You are asked to conduct the following experiment: You flip the first coin and note the result (heads or tails). If the first flip resulted in heads, you then flip the second coin and note the result. If, on the other hand, the first flip resulted in a tails, you then flip the third coin and note the result.
   (a) What is a reasonable sample space Ω for this experiment?
   (b) Let X be (the random variable giving) the number of heads which occurred during the experiment. What is the state space S_X of X?
   (c) If X is the same random variable as in the previous part and F_X is the cumulative distribution function (CDF) of X, what is $F_X(1.5)$?

4. A circuit has three nodes: A, B, and C. Each node is *independently* functional with probability p_A, p_B, and p_C respectively. The circuit works if either A is functional, or both B and C are functional. Otherwise, the circuit does not work. Find the probability that the circuit is working. (Your answer should be in terms of p_A, p_B, and p_C. This problem is not meant to be tricky!)

5. After watching Guardians of the Galaxy, our class became extremely motivated to befriend a raccoon. In this class there are 20 males and 16 females.

   (a) We are going to make two committees from the people in this class. The first committee will consist of 6 people, 3 males and 3 females, to search for a raccoon to befriend; the second committee will consist of 4 other people, 2 males and 2 females, to wait near a phone and medical supplies so that after the first committee actually finds a raccoon and realizes that they're vicious beasts, they can be bandaged up and an ambulance can be called. How many ways are there for us to create these committees?

   (b) After everyone survived the "raccoon incident" (and I lost my job), all 36 people in this class return to the classroom and shake everyone else's hands (no repeated handshakes) on a job well done. How many handshakes took place?

6. You... yes YOU!... have become sick of internet memes. Upon this realization, you decide to make your internet password using the letters MEMEOFF.

   (a) How many different passwords (distinguishable letter arrangements) can you make such that M is the first and fourth letter in the password?

   (b) Let A be the event that M is the first and fourth letter in the password, and let E be the event that both Es are next to each other in the password. If each password is equally likely, what is $P(E \mid A)$?

7. You conduct an "experiment" where you continually flip a coin (keeping track of the outcome of each flip) until the either the first heads appears or the coin has been flipped 5 times, at which point you stop.

   (a) What is a reasonable *sample space* for this experiment?

   (b) If E is the event that you flip the coin an odd number of times before stopping, describe the event E as a subset of the sample space. (For example: If a sample space is $\{1, 2, 3\}$ and F is the event that the outcome is even, then the description of F as a subset of $\{1, 2, 3\}$ is just $F = \{2\}$).

8. There are three urns: Urn A, Urn B, and Urn C. Urn A contains 4 red marbles and 6 blue marbles; Urn B contains 3 red marbles and 7 blue marbles; Urn C has 8 red marbles and 2 blue marbles. You are going to select a random marble from one of the urns; there is a $1/2$ probability you will draw the marble from Urn A; there is a $3/10$ probabilty you will draw a marble from Urn B; there is a $1/5$ probability you will draw the marble from Urn C.

   (a) Let R be the event that you select a red marble. Find $P(R)$.

   (b) Given that the marble you draw is red, what is the probability that you drew the marble from Urn A?

9. (a) How many distinguishable letter arrangements can be made from the word

   BEYONCE

   so that the Es are *not* next to each other?

   (b) (Jay-Z is lending g's...) Jay-Z has \$10 which he is going to distribute amongst his 3 friends. How many ways can he distribute the money among them if each is to receive at least \$1? We assume here that he will distribute in dollar increments.

10. An alien classroom has 29 students: 17 glargons and 12 bubuus. A committee of 2 students is to be selected.

    (a) For $k = 0, 1$, or 2, what is the number of ways to choose this committee if k of the students are to be bubuus? Note: You can either write out for each case $k = 0$, $k = 1$, and $k = 2$ explicitly, or you can instead just write one general formula in terms of k.

    (b) Suppose that each possible committee is equally likely. Let X be (the random variable giving) the number of bubuus selected for the committee. Find the probability mass function p_X of X.

11. You are lost wandering through a desert with no water and you come upon the a lovely sorceress named Beyonce. Beyonce has 3 bottles in a box to her left: Bottle #1 containing 1 liter of water, bottle #2 containing 2 liters of water, and bottle #3 containing 3 liters of water. Beyonce will ask you a first question which you have a 50% chance of getting correct. If you get the answer wrong then she disappears along with all her water bottles and you get nothing. If you get the answer correct, then you are given bottle #1 and she will ask you a second question which you have a 30% chance of getting correct. If you get the second question wrong then she disappears along with her remaining two bottles (you are left with only bottle #1 which you won in the first round). If you get the second question correct, then she gives you bottle #2 (so you now have bottles #1 and #2) and asks you a third question which you have a 10% chance of getting correct. If you get the third question wrong, then she disappears and takes the third bottle of water with her (you are left with bottles #1 and #2 which you won during the first two rounds). If you get the third question correct, then she gives you bottle #3 (so you now have bottles #1, #2, and #3) and she vanishes.

    (a) Let X be (the random variable giving) the *liters of water* that you receive in your encounter with Beyonce. What is a good and reasonable state space S_X for the random variable X?

    (b) Let E be the event that you received bottles #1 and #2, but not bottle #3. Find $P(E)$.

    (c) Let k be the value in the state space S_X such that $\{X = k\}$ is the same event as E (where X and E are the same as the in the previous two parts). What is k?

12. This probability class has 37 students in it: 24 males, 13 females. The search for Mothman was so successful, I decide to put together another search committee to look for the Skunk Ape (another cryptid rumored to live in the southern parts of the country). I'm going to choose 5 males and 5 females for the search committee. Two *male* students are feuding and refuse to be on the same search committee.

    (a) Selecting from the students in class, how many distinguishable search committees are possible?

    (b) Selecting from the students in class, how many distinguishable search committees are possible if, once I've selected the students to form the committee, I'm going to select two of the males and two of the females on the committee to be leaders, while the other six will be followers? Note: the feuding parties still refuse to be on the same search committee no matter what position they have.

13. Little Red is trying to get to her grandmother's house, but the Wolf is trying to stop her! To get to her grandmother's house, Little Red walks down a road until the road splits. The split forces her to either go left or right. If she goes to the left she must *independently* find either a cloak of invisibility or a sword (or both) to get past the Wolf and get to her grandmother's

house. If she goes right she must find the Wolf-Off! spray to get past the Wolf and get to her grandmother's house.

Assume that Little Red has a probability p_L of going left at the split in the road, and p_R of going right. Given that Little Red goes left at the split, she has a probability p_C to find the cloak of invisibility and probability p_S to find the sword. Given that she goes right at the split, then she has a probability of p_W of finding the Wolf-Off! spray.

Question: What is the probability that Little Red makes it to her grandmother's house?

14. The following parts refer to the letters: LAMEFIREALARM. Recall that "word" means *distinguishable* letter arrangements.
    (a) How many words can be made with the above letters such that the M's are *not* next to each other?
    (b) What is the probability that, given a random word made from the letters above, the M's will be separated by at least two letters?

15. Suppose you shuffle together two standard 52 card decks (effectively, you double the number of each card). You are randomly dealt a 5 card poker hand. Let X be the random variable counting the number of 2's in your hand.
    (a) What is the state space S_X of X?
    (b) What is the probability mass function p_X of X? That is, find $p_X(k) = P(X = k)$ for k in S_X.
    (c) What is the value of the distribution F_X of X at 5.1? That is, find $F_X(5.1) = P(X \leq 5.1)$.

16. Marble 1, marble 2, and marble 3 are in an urn. You select one of the three marbles out of the urn in such a way that you have a 40% chance of selecting marble 1, a 50% chance of selecting marble 2, and a measly 10% chance of selecting marble 3. After you've selected the marble you then roll fair dice in the following way: if you selected marble 1, you roll one fair die; if you selected marble 2, you roll two fair dice; if you selected marble 3, you roll three fair dice. Given that the sum of the outcomes of the dice you roll is 3, what is the probability that you selected marble 2? (To make sure we're clear: If you roll three dice and the outcomes are i, j, and k, then the sum of the outcomes is $i + j + k$. If you roll two dice and the outcomes are i and j, then the sum of the outcomes is $i + j$. If you roll one die and the outcome is i, then the sum of the outcomes is i.)