PROBABILITY

Math 3160Q – Spring 2015
Professor Hohn

Exercises that are fair game for Thursdays quizzes/exams.

Quiz 1 – Thursday, January 22

- No exercises.

Quiz 2 – Thursday, January 29

- Chapter 1: Problems (pg 15ff): 8, 10, 11, 13, 16, 18, 25, 27, 28, 30
- Chapter 1: Theoretical Exercises (pg 17ff): 8 (see note below), 13, 16, 17 (see note below)
- Note: For theoretical exercise #8, interpret "prove" as "give a convincing argument." For theoretical exercise #17, interpret "combinatorial argument" as "explain why you expect this to be true by a counting example, rather than just the mathematical algebra."

Quiz 3 – Thursday, February 12

- Chapter 2: Problems (pg 48ff): 2, 4, 8, 9, 12, 15, 19, 23, 25, 31, 36, 38, 41, 43, 45
- Chapter 2: Theoretical Exercises (pg 52ff): 9, 11, 13

Quiz 4 – Thursday, February 19

- Chapter 3: Problems (pg 97ff): 1, 4, 5, 11, 14, 17, 24, 27, 29, 32, 39, 40, 44
- Chapter 3: Theoretical Exercises (pg. 106ff): 2, 5

Exam 1 – Thursday, February 26

- Chapter 3: Problems (pg 97ff): 41, 51, 53, 54, 66, 70, 81
- Chapter 4: Problems (pg. 163ff): 1, 2, 5 (see note below), 7 (see note below), 13
- Note: For #5 and #7, when the book says, "what are the possible outcomes..." for a particular random variable, I want you to read it as, "what is the state space of ..." that particular random variable.

Exam 1 Study Guide/Tips: All questions listed above are fair game. In addition, any quiz, worksheet, or example problem has a chance of being on the exam. I also strongly suggest you work through the review questions at the end of each chapter as well.

Quiz 5 – Thursday, March 5

- Chapter 4: Problems (pg 163ff): 17, 18, 19, 20(a), 20(b), 40 (see note below), 60
- Note: For #40, you can likely answer this without the "technology" of random variables; however, try and use the random variable technology to get used to it. In particular, if X is the (random variable giving the) number of correct answers the student gets while guessing, then convince yourself that $X \overset{d}{=} \text{Bin}(n, p)$ for a good choice of n and a good choice of p.

Quiz 6 – Thursday, March 12

- Chapter 4: Problems (pg 163ff): 20c, 25, 30, 32, 33, 35, 37, 38, 41, 51, 53, 58
- Chapter 4: Theoretical Exercises (pg. 169ff): 3, 6, 19
- Not from Ross:
  1. Let X be a random variable with CDF given by
  $$F_X(t) = \begin{cases} 0 & t < -1 \\ \frac{1}{2} & -1 \leq t < 1 \\ \frac{1}{2}t & 1 \leq t < 2 \\ 1 & t \geq 2 \end{cases}$$
  Calculate $\mathbb{E}[X]$.
  2. Suppose that the number of earthquakes per year in a certain region in the US is well-modeled by a Poisson random variable with an average of 3 earthquakes occurring per year. Calculate the probability that in a given year there are at least 4 earthquakes in this region, given that there are at least 2 earthquakes.

Quiz 7 – Thursday, March 26

- Chapter 4: Problems (pg 163ff): 52, 54, 55, 57, 72, 79
- Chapter 4: Theoretical Exercises (pg. 169ff): 20
- Chapter 5: Problems (pg 212ff): 1, 2, 3, 4, 7, 10, 11, 12, 13
- Not from Ross:
  1. Let $X \overset{d}{=} \text{Bin}(4, 1/3)$. What is $P(X^2 + X \geq 1)$?
  2. Let $X \overset{d}{=} \text{Bin}(4, 1/3)$ and $Y \overset{d}{=} \text{Geom}(1/2)$. For each choice of Z, find the state space S_Z of Z and calculate $\mathbb{E}[Z]$:
     (a) $Z = Y - X$.
     (b) $Z = X^2 + 3Y$.

Quiz 8 – Thursday, April 2

- Chapter 5: Problems (pg 212ff): 15 (see note below), 17, 18 (see note below), 19 (see note below), 21 (see note below), 23, 25, 26, 27, 28, 32, 33, 40, 41
- Chapter 5: Theoretical Exercises (pg 214ff): 1, 9 (see note below), 10, 15, 31
- Not from Ross:

1. Use the differentiation trick

$$\int_0^\infty t^n e^{-\lambda t}\,dt = (-1)^n \left(\frac{\partial}{\partial \lambda}\right)^n \int_0^\infty e^{-\lambda t}\,dt$$

to find a general formula (depending on n) for $\int_0^\infty t^n\, e^{-\lambda t}\,dt$. Note that $\left(\frac{\partial}{\partial \lambda}\right)^n$ is just notation meaning the nth derivative with respect to λ; another way this is commonly written is $\frac{\partial^n}{\partial \lambda^n}$. Use this to find $\mathbb{E}[X^5]$ where $X \overset{d}{=} \mathrm{Exp}(2)$.

2. Recall that we showed in class

$$\int_{-\infty}^\infty e^{-t^2/2}\,dt = \sqrt{2\pi}\,.$$

Use the same idea to show that

$$\int_{-\infty}^\infty e^{-\alpha\, t^2}\,dt = \sqrt{\frac{\pi}{\alpha}}\,.$$

3. Use the differentiation trick

$$\int_{-\infty}^\infty t^{2n} e^{-\alpha t^2}\,dt = (-1)^n \left(\frac{\partial}{\partial \alpha}\right)^n \int_{-\infty}^\infty e^{-\alpha t^2}\,dt$$

to find a general formula (depending on n) for $\int_{-\infty}^\infty t^{2n}\, e^{-\alpha t^2}\,dt$. Note that $\left(\frac{\partial}{\partial \alpha}\right)^n$ is just notation meaning the nth derivative with respect to α; another way this is commonly written is $\frac{\partial^n}{\partial \alpha^n}$. Use this to find $\mathbb{E}[Z^4]$ where Z is a standard normal random variable.

Notational Hint: Notice that if you multiply together all the odd numbers $1{\cdot}3{\cdot}5\cdots(2N-1)$, you can rewrite this as $\prod_{k=0}^{N-1}(1+2k)$. However, there is also a commonly accepted "double factorial" notation $(2N-1)!!$ which means exactly the product of all the positive odd numbers up to $2N-1$ (the double factorial also works for even numbers if the number you write the double factorial against is even). So,

$$1 \cdot 3 \cdot 5 \cdots (2N-1) = \prod_{k=0}^{N-1}(1+2k) = (2N-1)!!.$$

- Note:
  - For both #15 and #17, leave your answer in terms of Φ. For the second question in #21, it is asking for you to condition on the event that the height is ≥ 6 feet.
  - The CDF Φ of a standard normal random variable is invertible on the range $(0,1)$. That is, if for some $0 < y < 1$ you have an expression of the form $\Phi(x) = y$, you can solve for x by $x = \Phi^{-1}(y)$ (where Φ^{-1} is the inverse of Φ, **not** $1/\Phi$). Therefore, if you don't want to use the table of values for Problems #18 and #19, you can leave your answer in terms of the inverse Φ^{-1}.
  - For Theoretical Exercise #9, when the book says "show," what I want you to do is to draw a sketch of the density of Z and convince yourself that the equalities are true by shading in the appropriate areas.

Exam 2 – Thursday, April 9

- Chapter 6: Problems (pg 271ff): 2, 4,
- Do all of Exam 2 Practice Problems

Exam 2 Study Guide/Tips: All questions listed above are fair game. In addition, any quiz, worksheet, or example problem has a chance of being on the exam. I also strongly suggest you work through the review questions at the end of each chapter as well.

Quiz 9 – Thursday, April 16

- No new exercises: Exam recovery!
- Chapter 6: Problems (pg 271ff): 2, 4,

Quiz 10 – Thursday, April 23

- Chapter 6: Problems (pg 271ff): 8, 9, 10, 13, 14, 15, 20, 23, 26, 27
- Not from Ross:
    1. Suppose that X and Y are jointly distributed and discrete with joint mass function

    $$p_{X,Y}(s,t) = \begin{cases} c\,(s+t) & s \in \{2,3,4,5\}, t \in \{1,...,s\} \\ 0 & \text{otherwise} \end{cases}$$

    Find c and justify whether or not X and Y are independent.

Quiz 11 – Thursday, April 30

- Chapter 6: Problems (pg 271ff): 28, 38, 45, 56
- Chapter 7: Problems (pg 352ff): 4, 12, 16, 30, 37, 45, 75
- Chapter 7: Theoretical Exercises (pg 359ff): 19
- Not from Ross: All exercises in the Covariance and Correlation Notes

Final Exam

- Chapter 8: Problems (pg 390ff): 3, 4b, 14, 15
- Chapter 4: Problems (pg 163ff): 70
- Do all of Final Exam Practice Problems

Final Exam Study Guide/Tips: All questions listed above are fair game. In addition, any quiz, worksheet, or example problem has a chance of being on the exam. I also strongly suggest you work through the review questions at the end of each chapter as well.